Lecture

Tessellations, fractals, projection

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Advanced Topics in Digital Design

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Outline

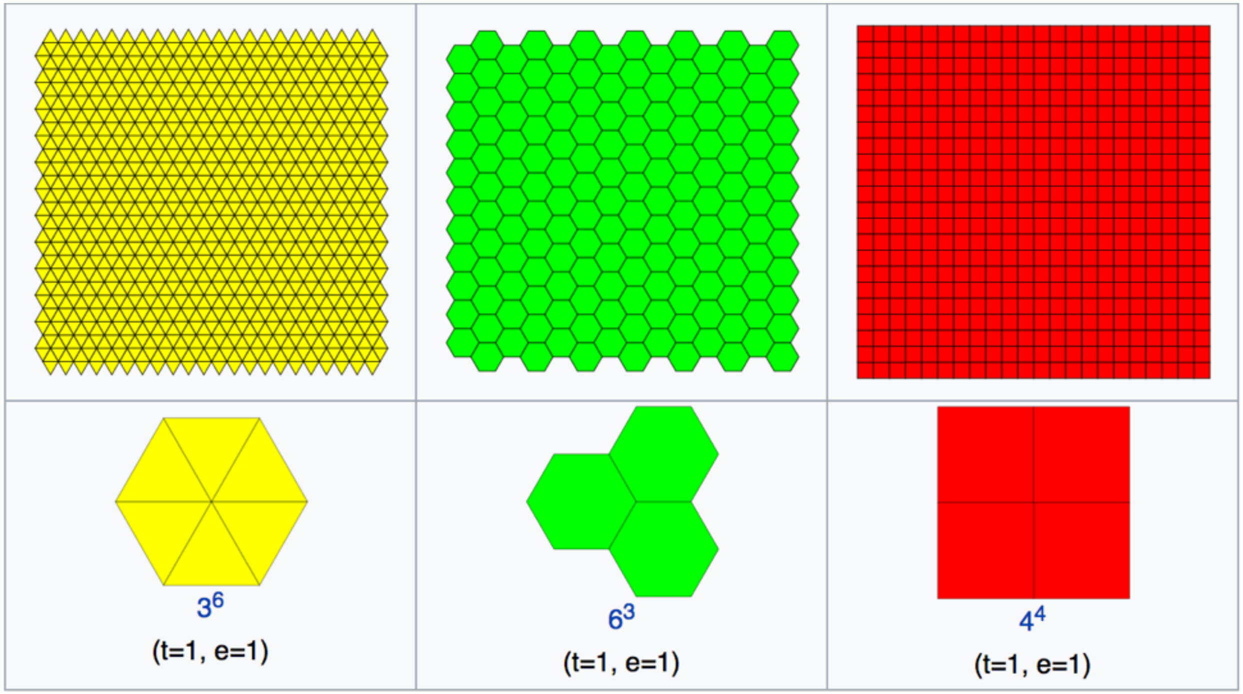
- Tessellations
- Circle packing
- Voronoi Diagram & Delaunay Triangulation
- Fractals
- Projection

Online sources

Tessellations

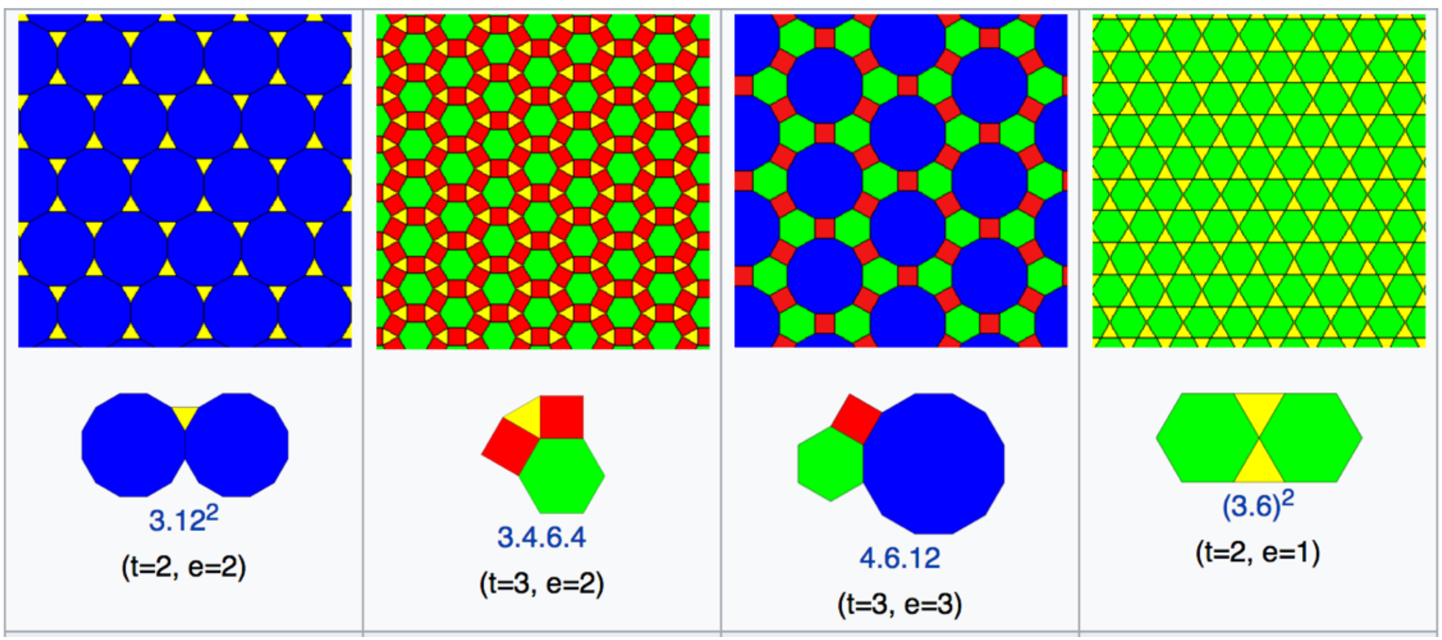
Euclidean Tilings by Convex Regular Polygons

- **Regular Tilings** has one type of regular face.



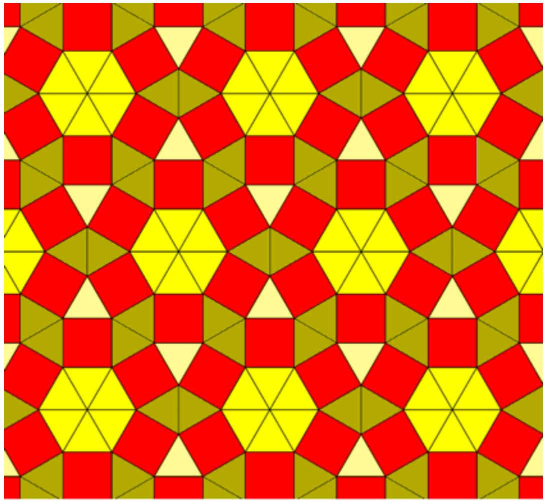
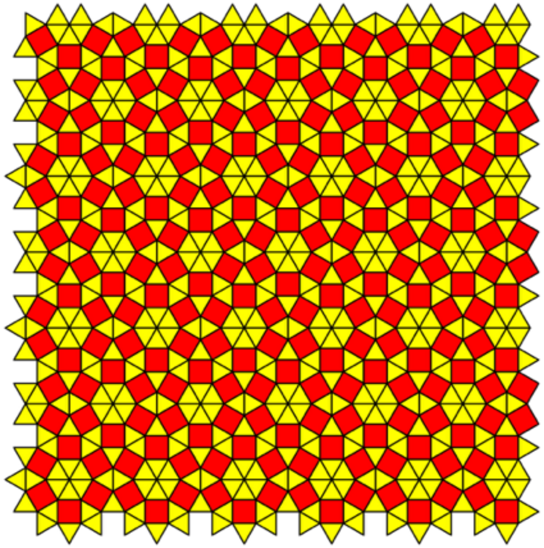
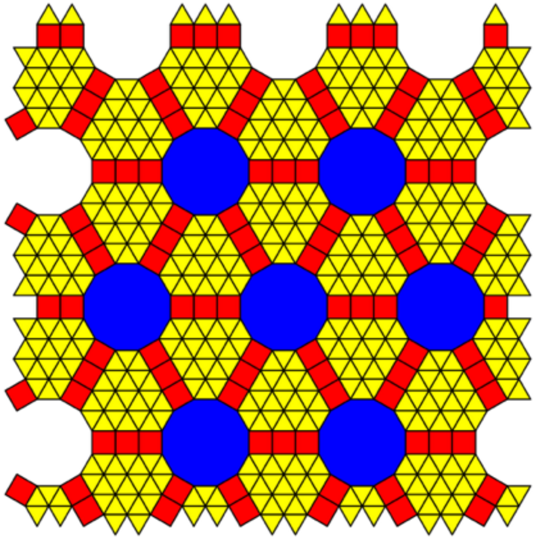
Euclidean Tilings by Convex Regular Polygons

- **A regular tiling** has one type of regular face.
- **A semiregular or uniform tiling** has one type of vertex, but two or more types of faces.



Euclidean Tilings by Convex Regular Polygons

- **A regular tiling** has one type of regular face.
- **A semiregular or uniform tiling** has one type of vertex, but two or more types of faces.
- **A k-uniform tiling** has k types of vertices, and two or more types of regular faces.

Euclidean Tilings by Convex Regular Polygons

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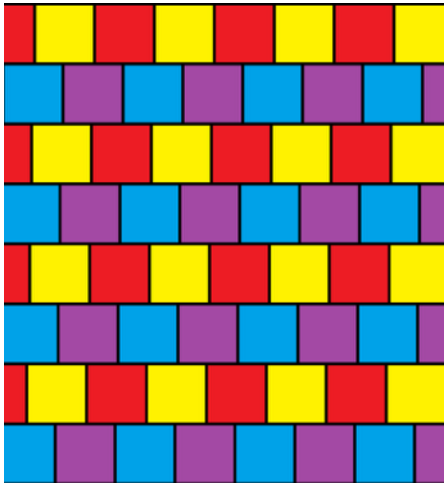
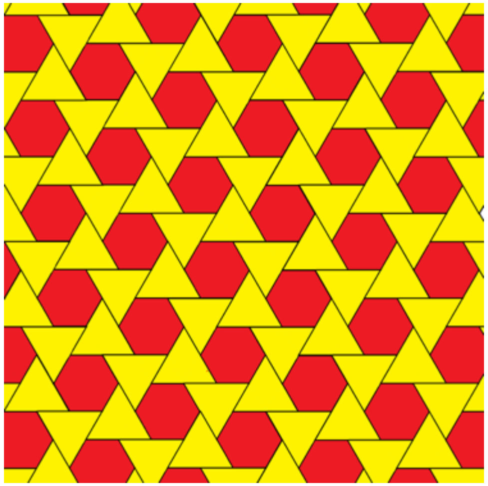
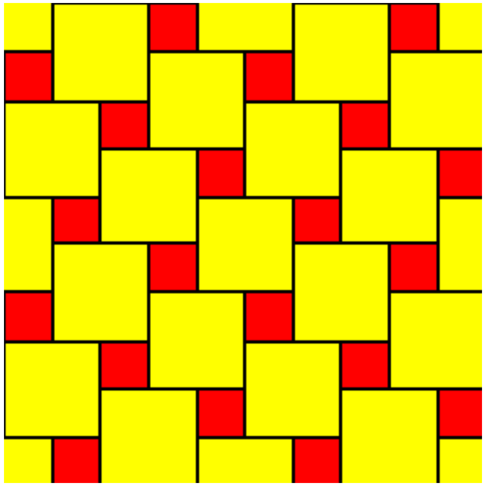
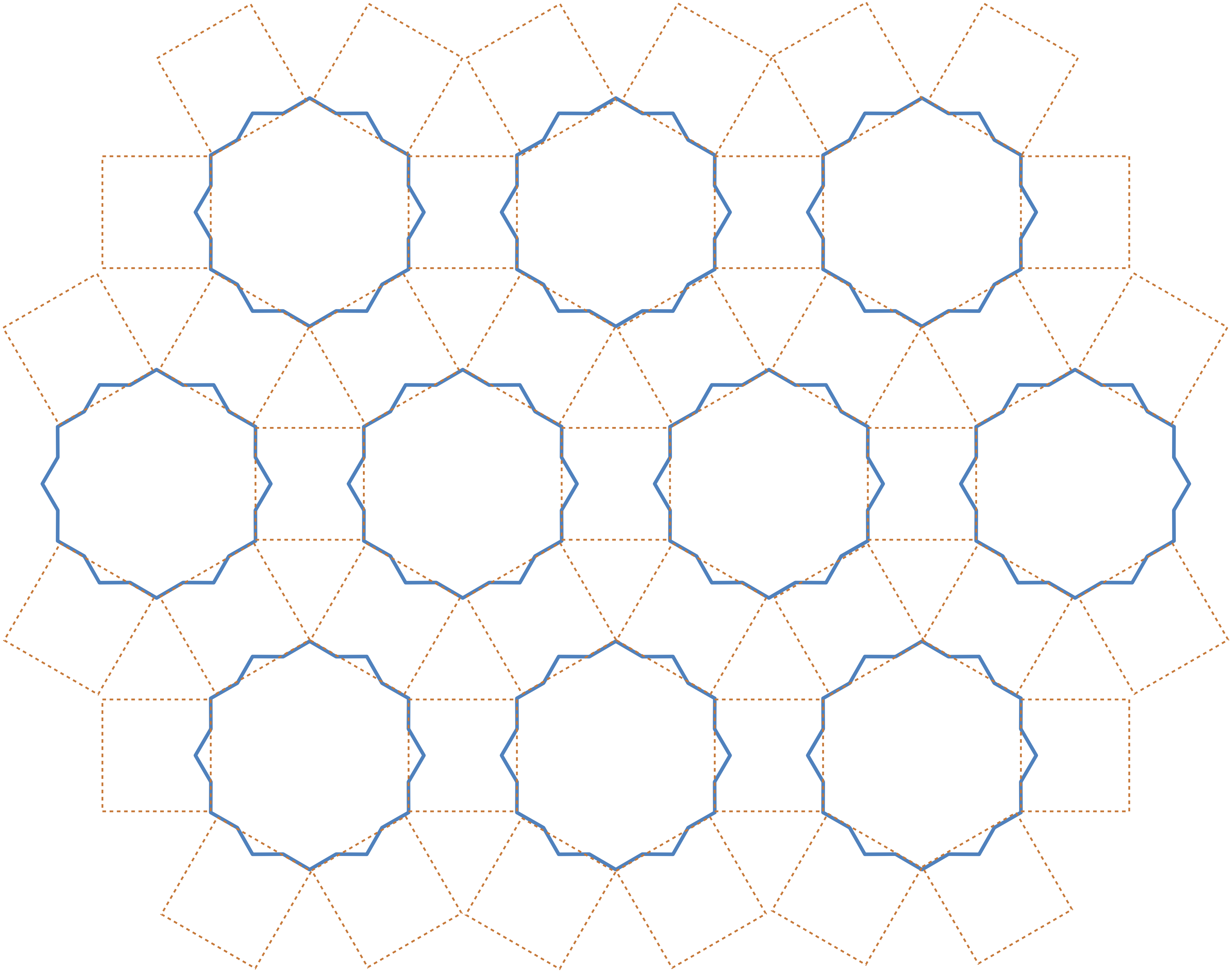
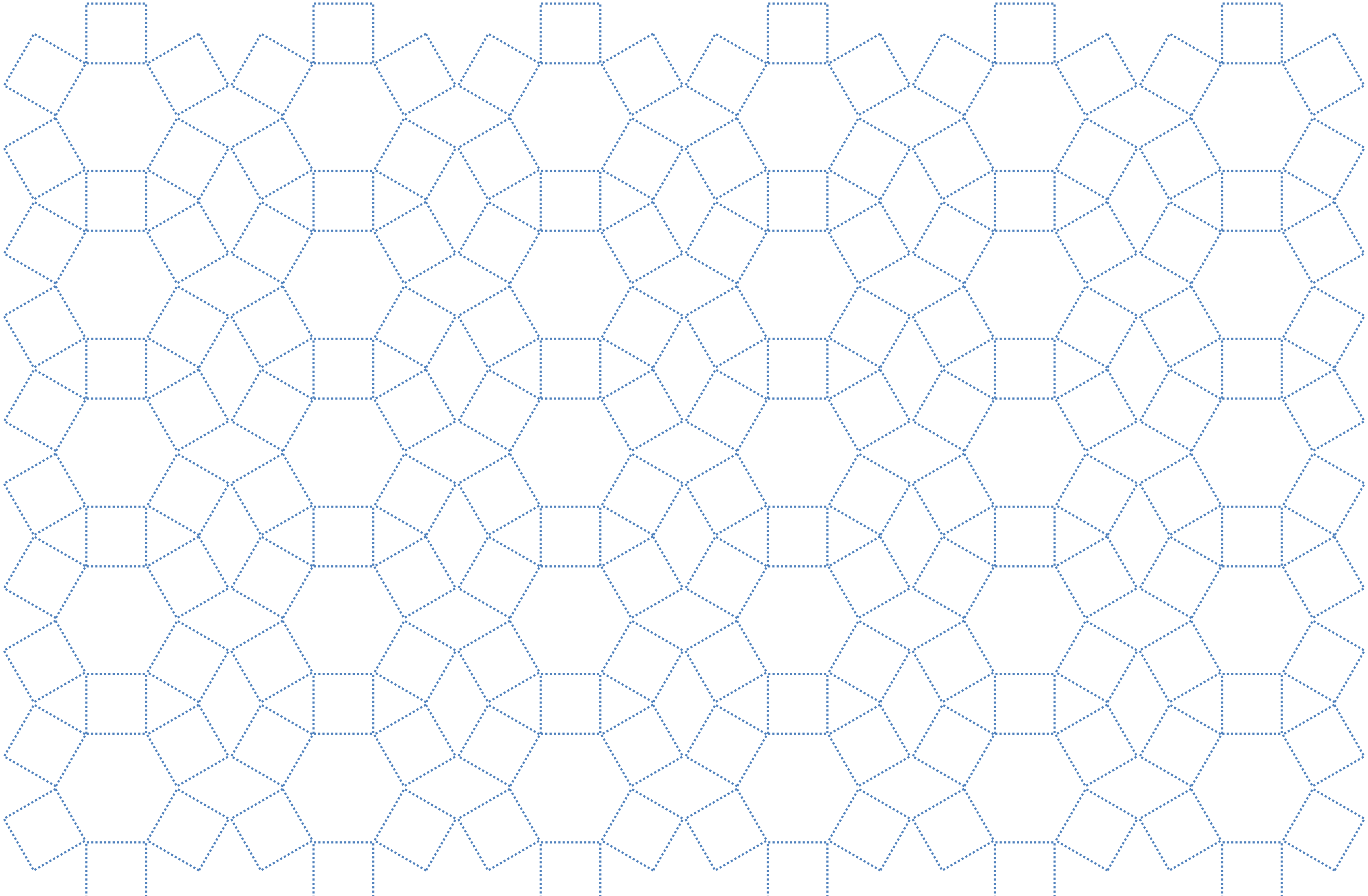
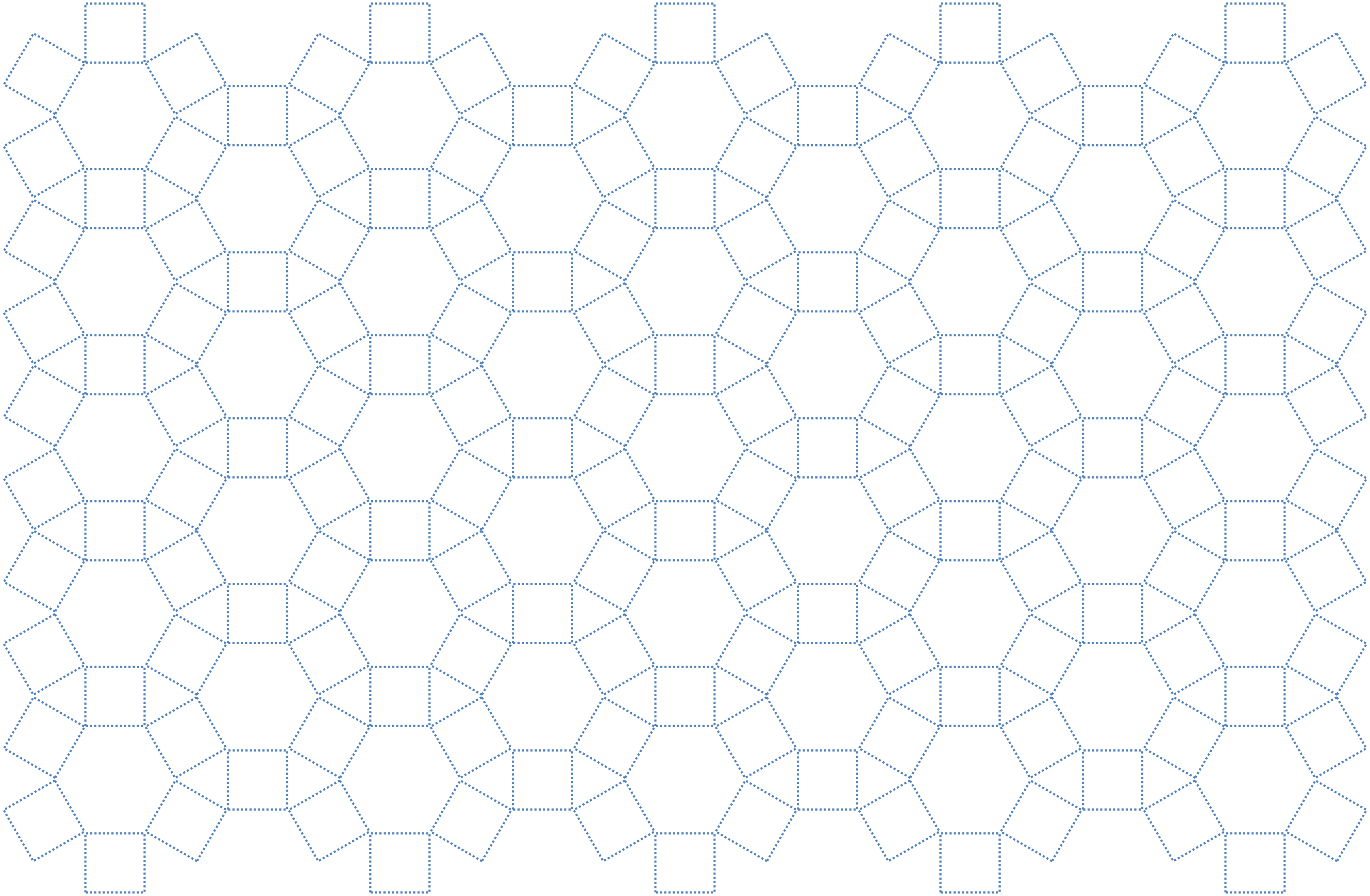
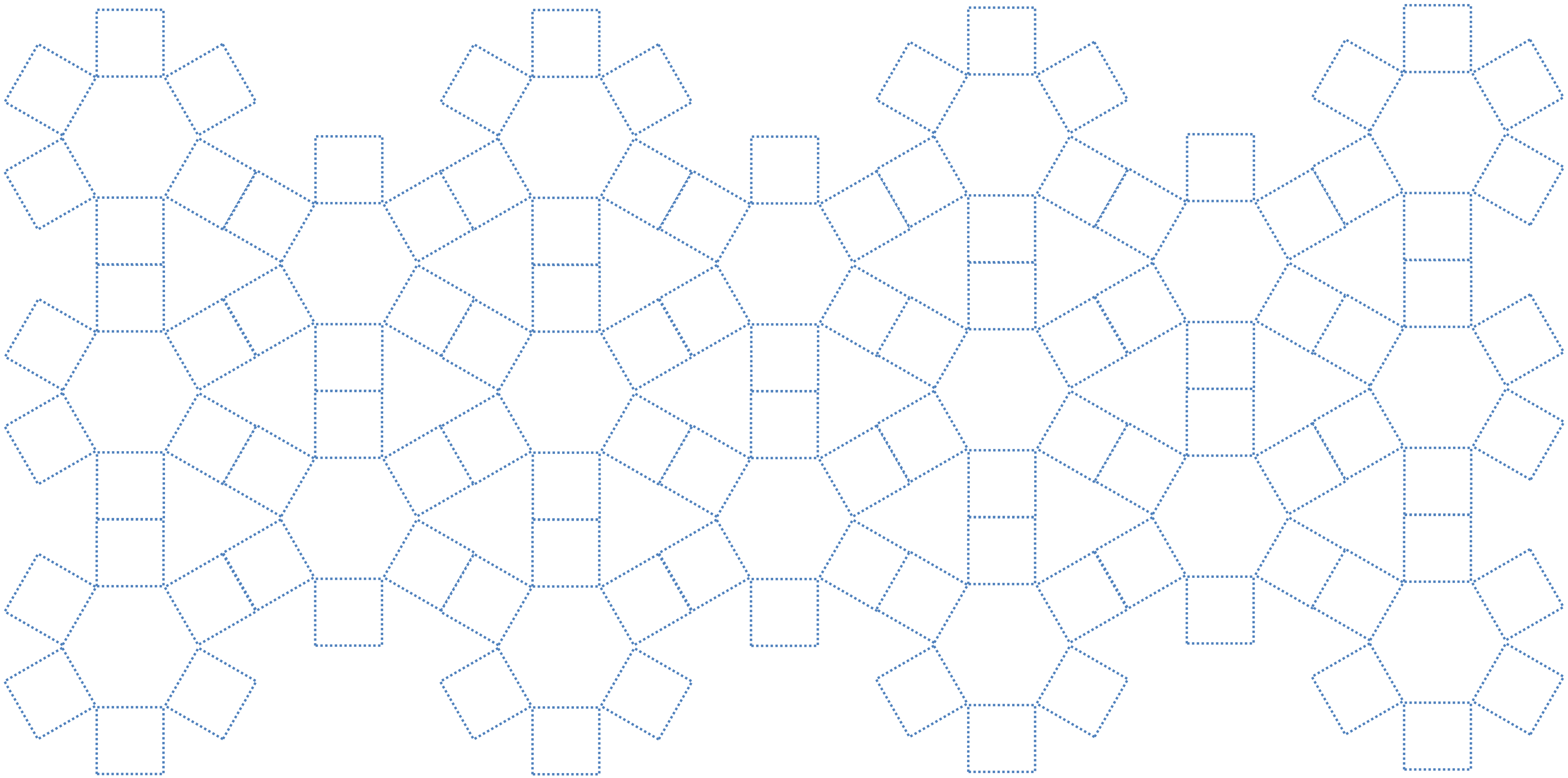
- **A semiregular or uniform tiling** has one type of vertex, but two or more types of faces.
- **A k-uniform tiling** has k types of vertices, and two or more types of regular faces.
- Tilings that are not edge-to-edge

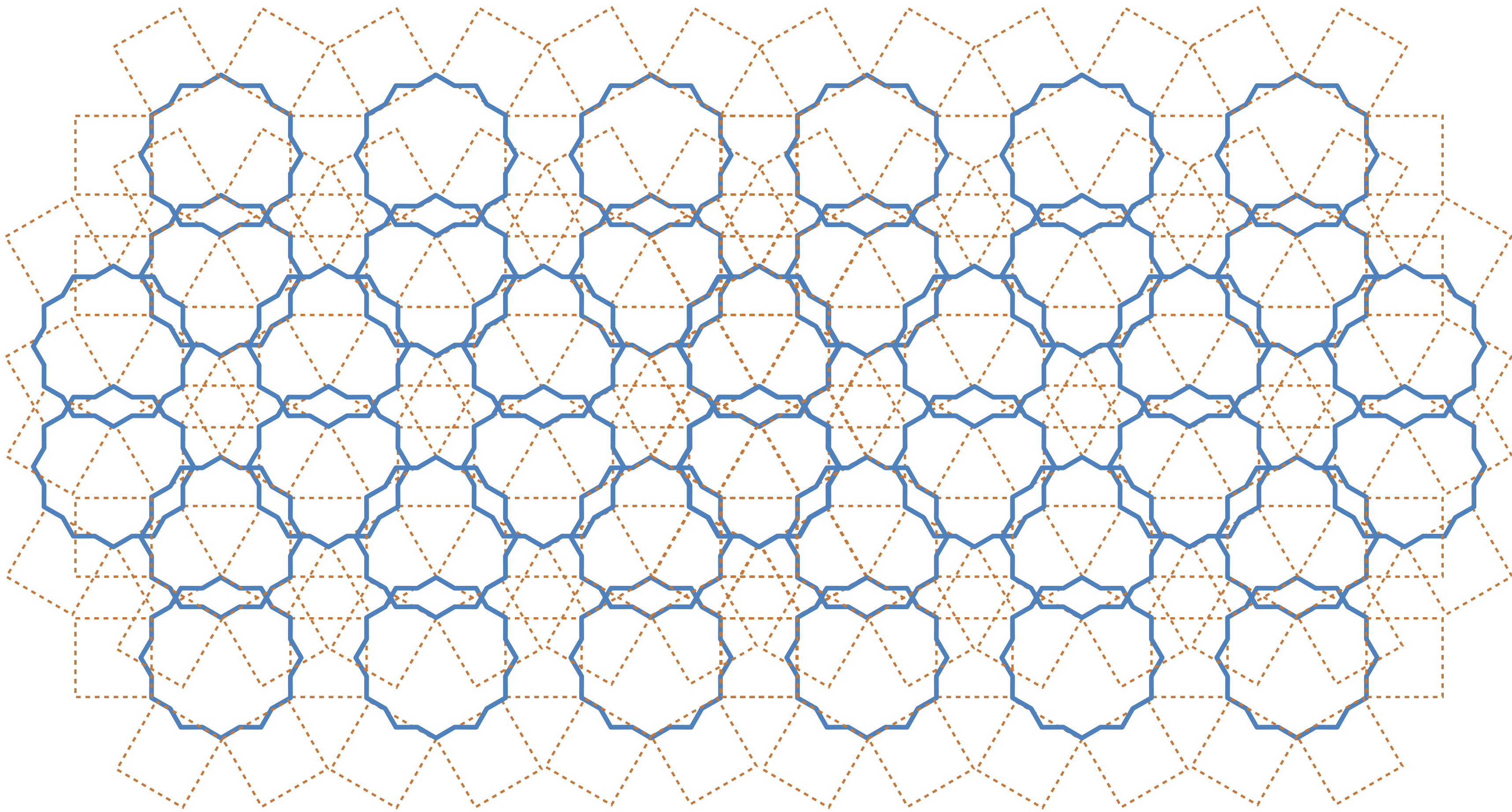






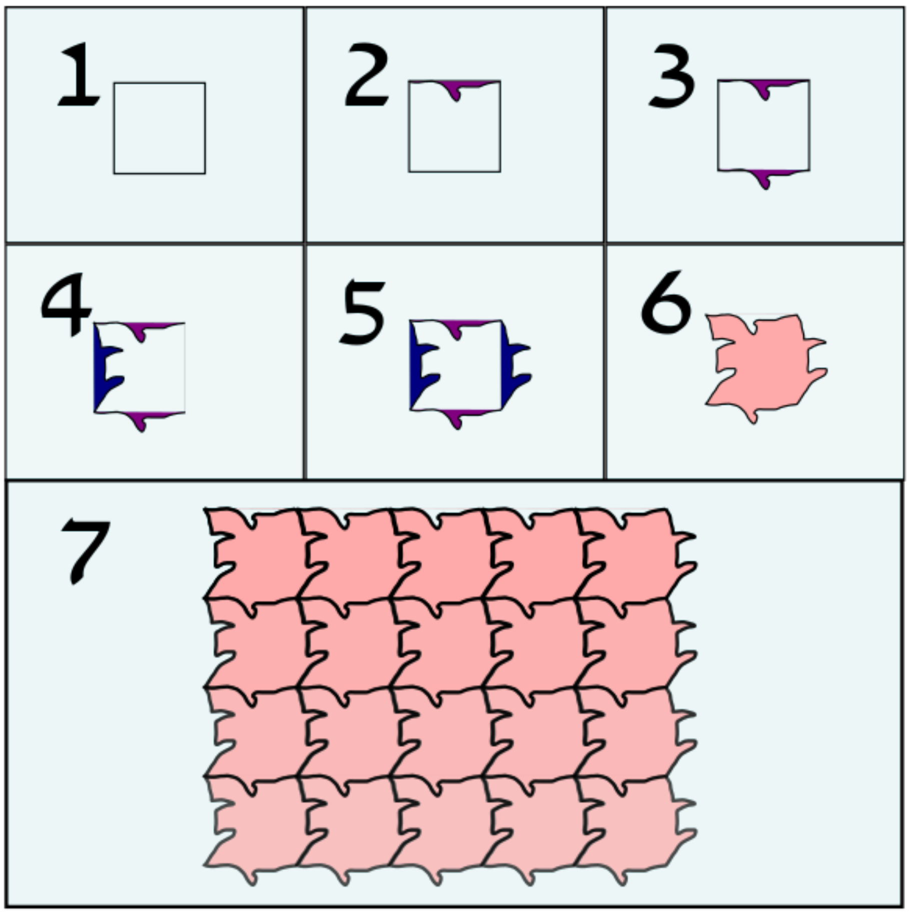








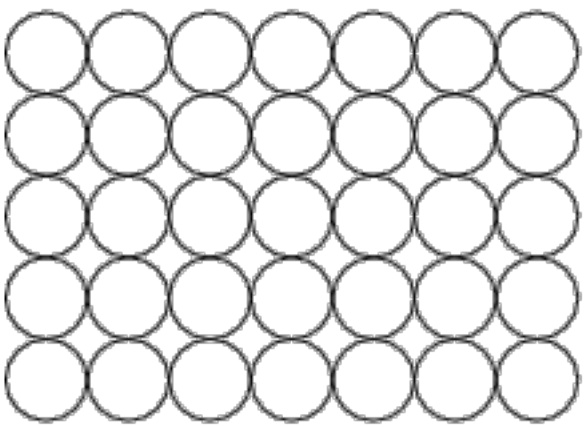
M. C. Escher tessellation style



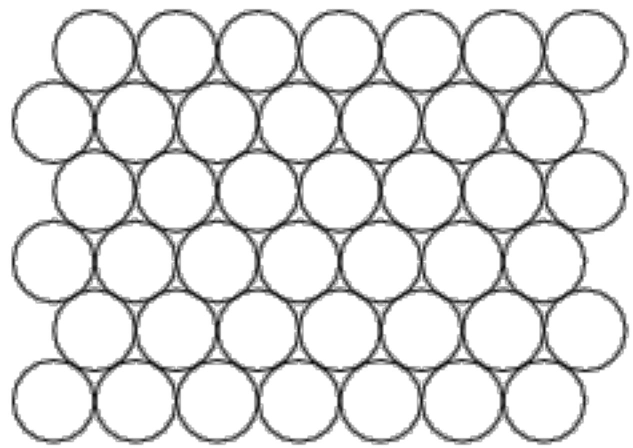
Circle Packing

Circle Packing

A circle packing is an arrangement of circles inside a given boundary such that no two overlap and some (or all) of them are mutually tangent. The generalization to spheres is called a sphere packing. Tessellations of regular polygons correspond to particular circle packings.



square packing

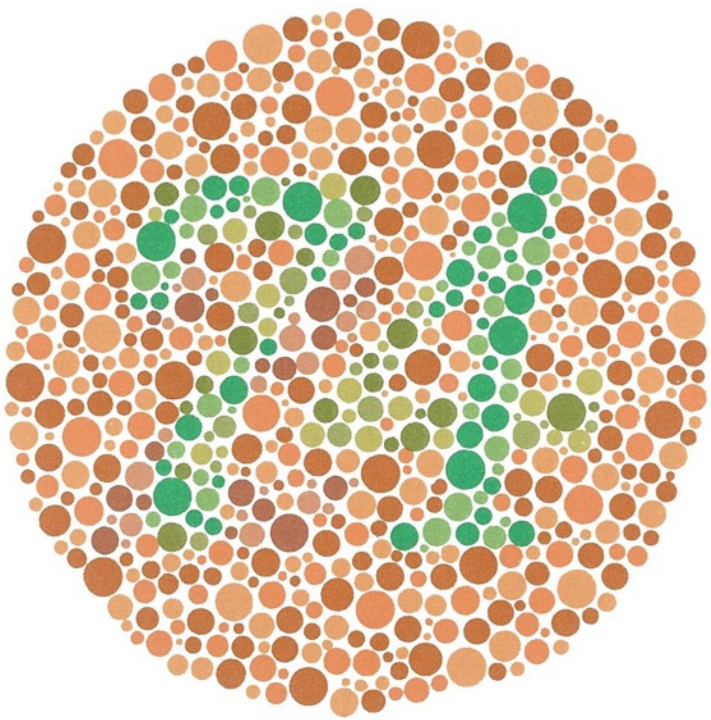


hexagonal packing

http://mathworld.wolfram.com/CirclePacking.html

Circle Packing

Problem: Position N circles of different radii inside a larger circle without overlapping => NP-hard, or use faster (polynomial time) estimation algorithm



http://stackoverflow.com/questions/3851668/position-n-circles-of-different-radii-inside-a-larger-circle-without-overlapping

https://www.ic.unicamp.br/~fkm/lectures/circle-packing.pdf

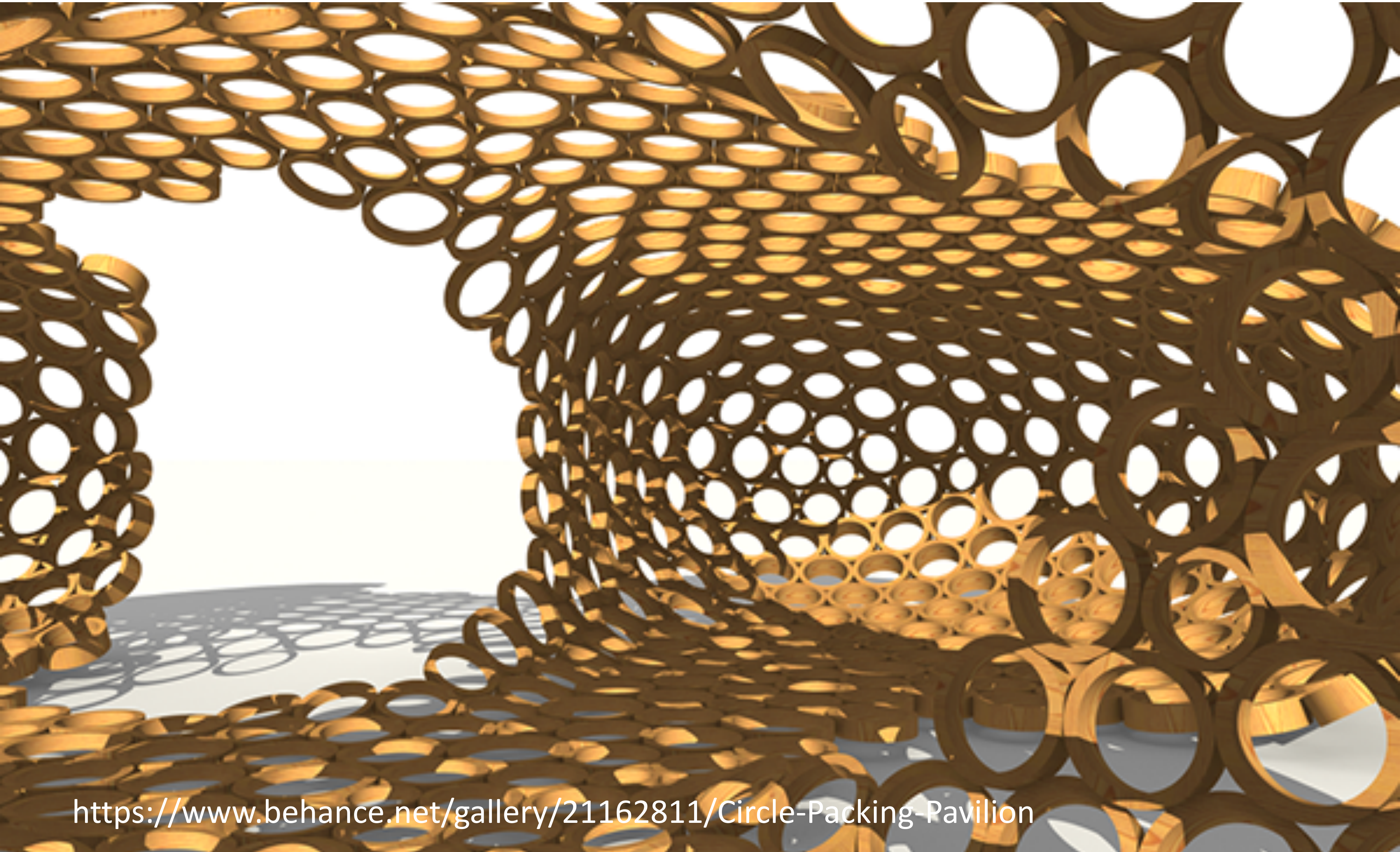
Circle Packing - applications

http://matsysdesign.com/wp-content/uploads/2009/06/constellation_06_circ.jpg

Circle Packing - applications

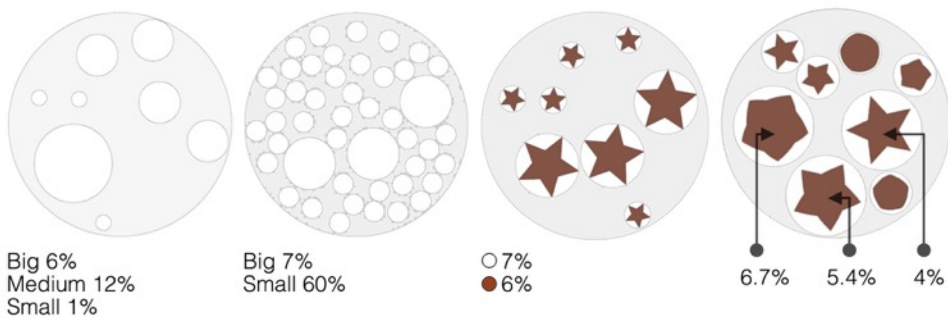
http://inhabitat.com/packed-pavilion-constructed-completely-from-cardboard/

Circle Packing - applications



https://www.behance.net/gallery/21162811/Circle-Packing-Pavilion

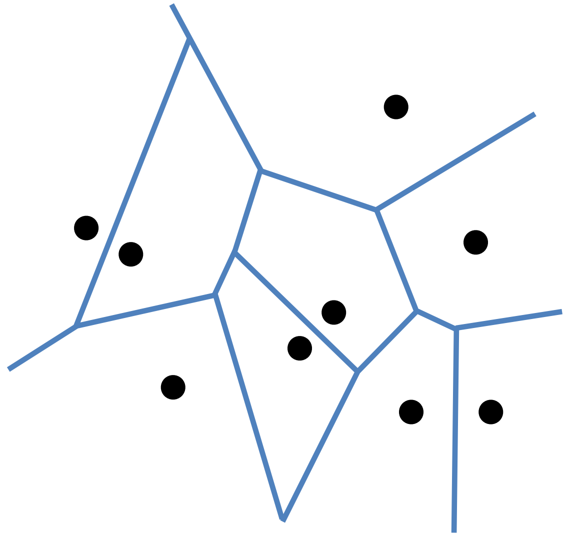
Circle Packing - applications



Voronoi Diagram & Delaunay Triangulation

Voronoi Diagram

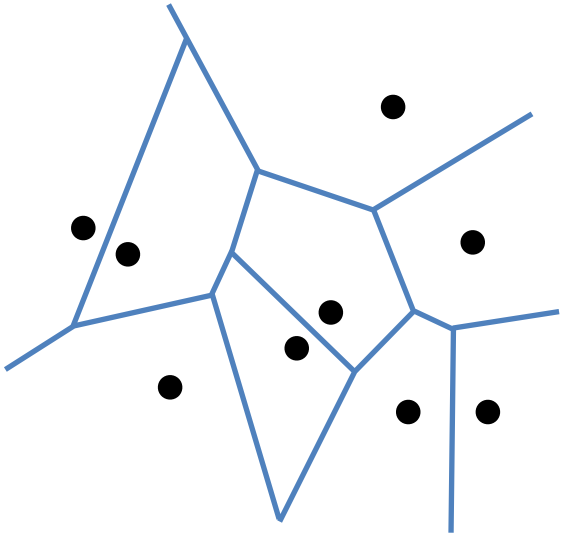
A Voronoi diagram is a partitioning of a plane into regions based on distance to points in a specific subset of the plane. That set of points (called seeds, sites, or generators) is specified beforehand, and for each seed there is a corresponding region consisting of all points closer to that seed than to any other. These regions are called Voronoi cells. The Voronoi diagram of a set of points is dual to its Delaunay triangulation.



Voronoi Diagram - formal definition

Let X be a metric space with distance function d. Let K be a set of indices and let $(P_k)_{k\in K}$ be an ordered collection of nonempty subsets (the sites) in the space X. The Voronoi cell R_k, associated with the site P_k is the set of all points in X whose distance to P_k is not greater than their distance to the other sites P_j, where j is any index different from k. In other words, if $d(x,A)=\inf\{d(x,a)|a\in A\}$ denotes the distance between the point x and the subset A, then

$$R_k = \{x \in X \mid d(x, P_k) \leq d(x, P_j) \text{ for all } j \neq k\}$$



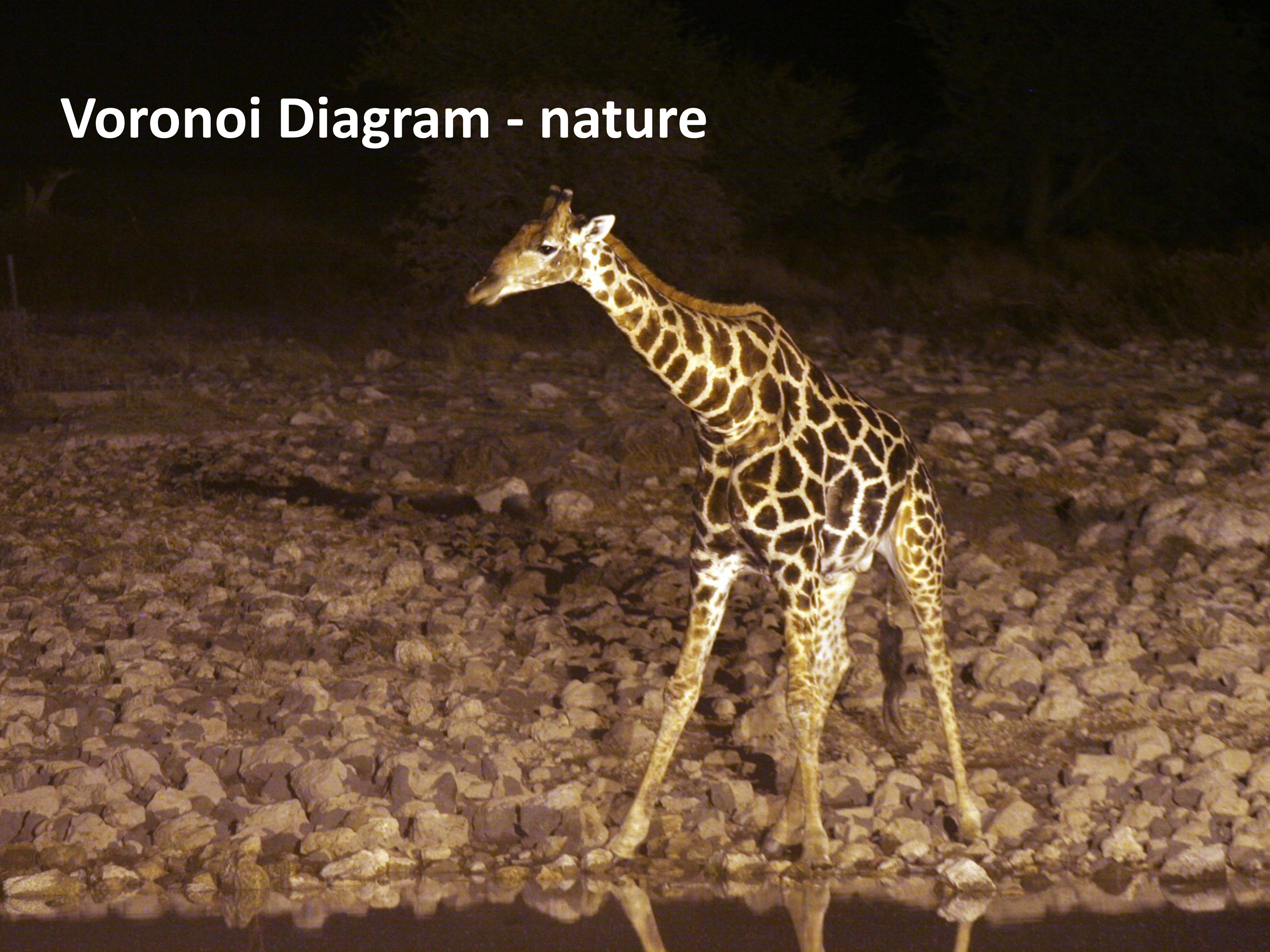
Voronoi Diagram - some properties

- The closest pair of points corresponds to two adjacent cells in the Voronoi diagram.
- Under relatively general conditions Voronoi cells enjoy a certain stability property: a small change in the shapes of the sites, e.g., a change caused by some translation or distortion, yields a small change in the shape of the Voronoi cells. This is the geometric stability of Voronoi diagrams.

Voronoi Diagram - Fortune's algorithm

O($n$log$n$)

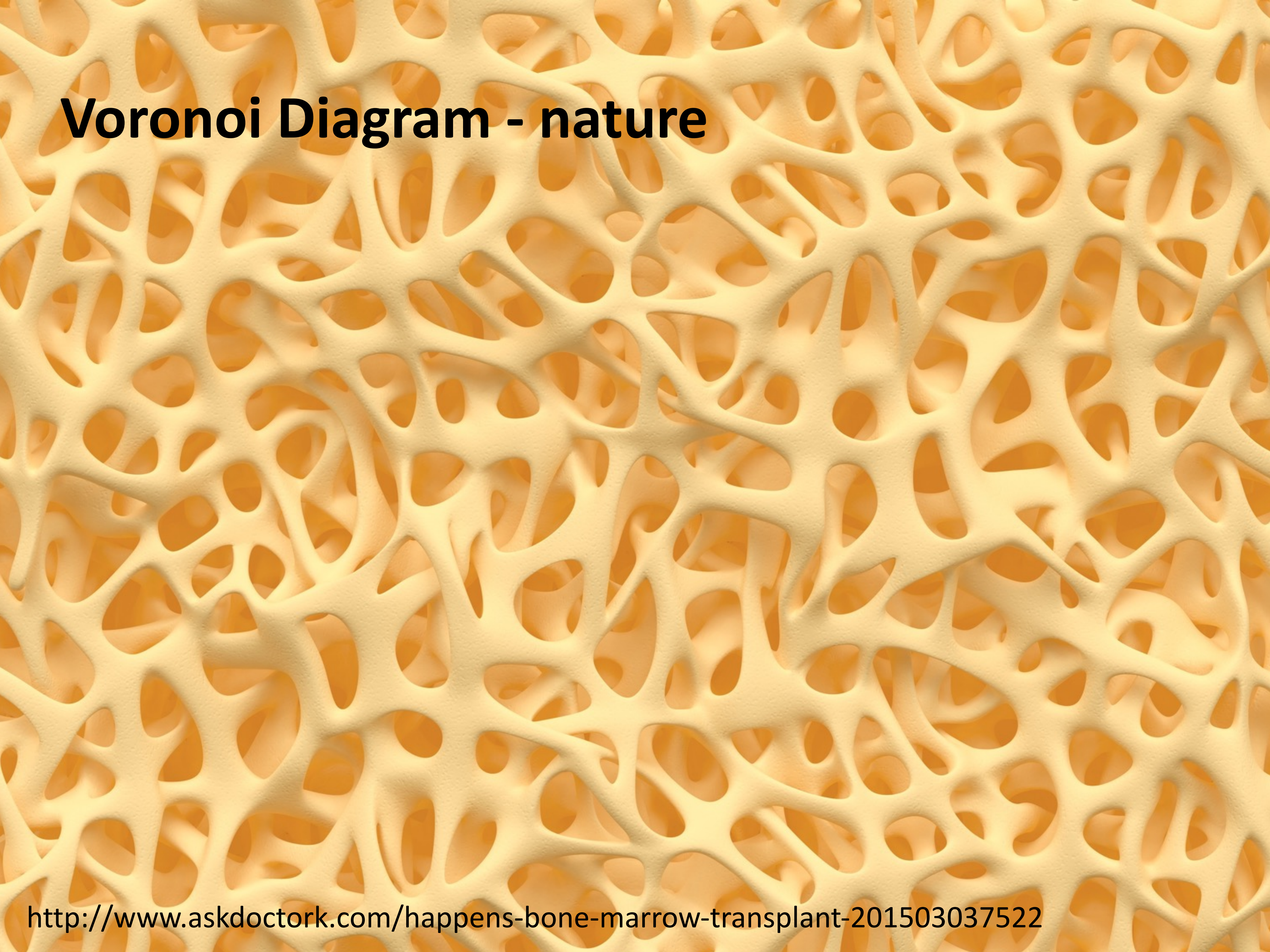
Voronoi Diagram - nature



Voronoi Diagram - nature

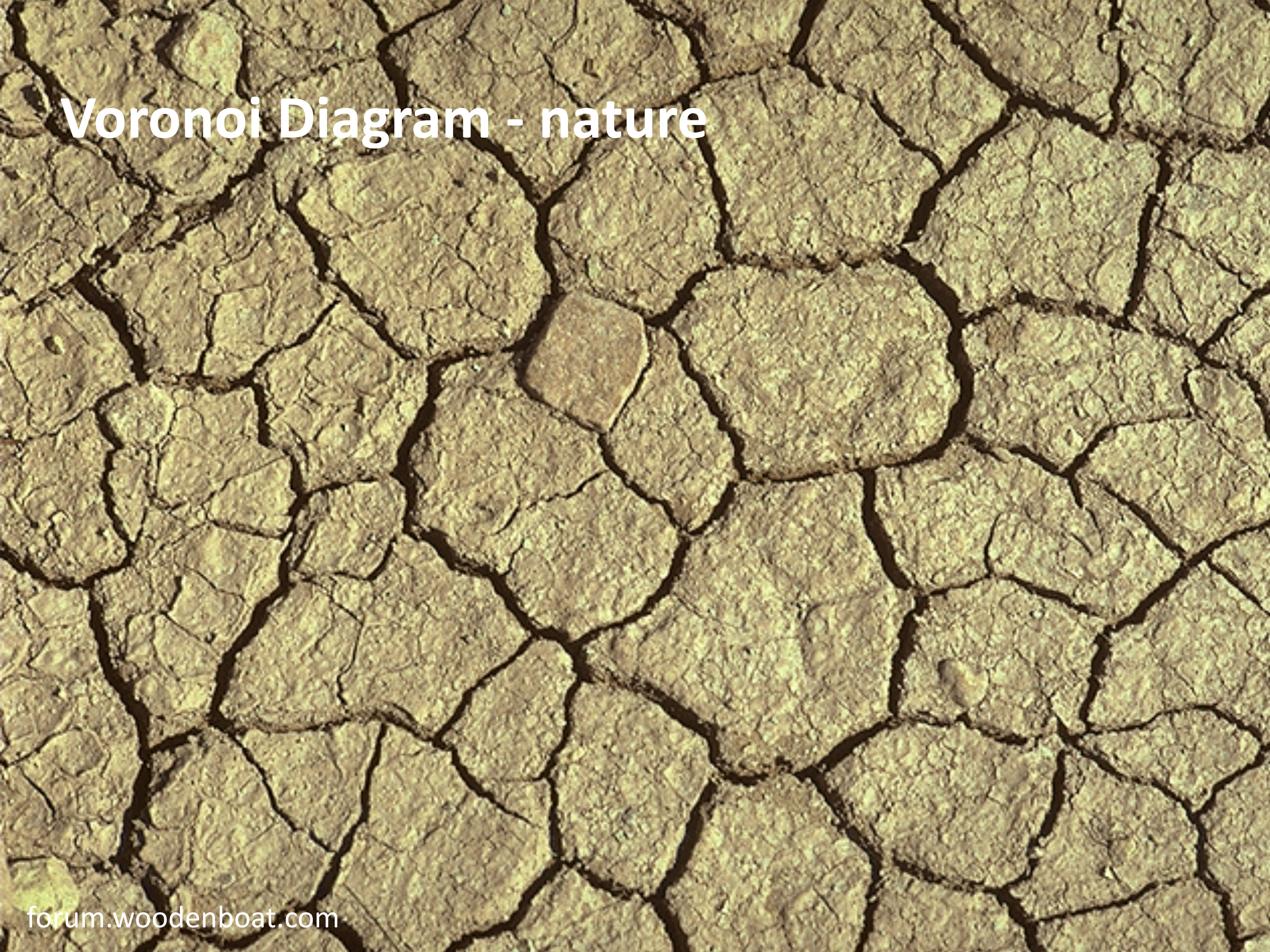
http://www.pbase.com/rcm1840/image/45007220

Voronoi Diagram - nature



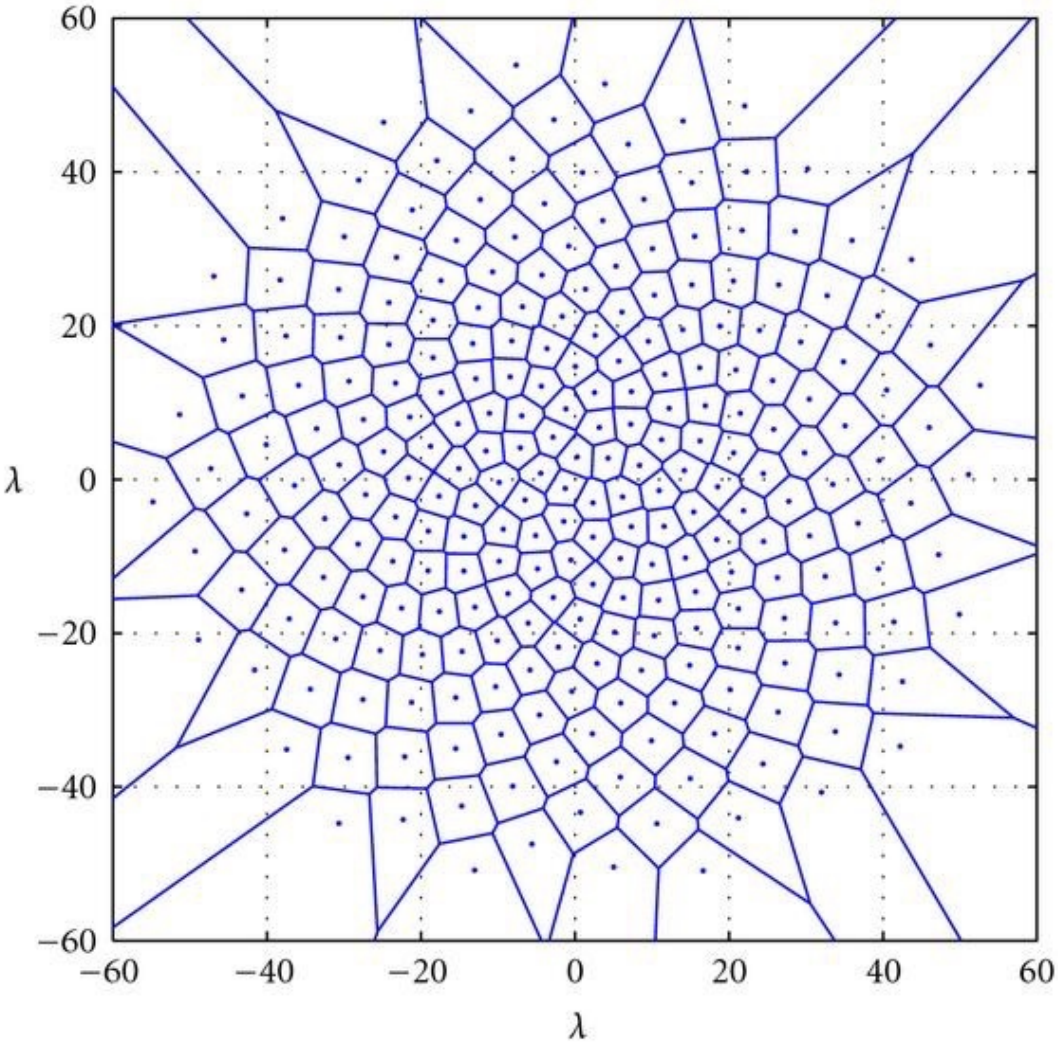
http://www.askdoctork.com/happens-bone-marrow-transplant-201503037522

Voronoi Diagram - nature



forum.woodenboat.com

Voronoi Diagram - nature



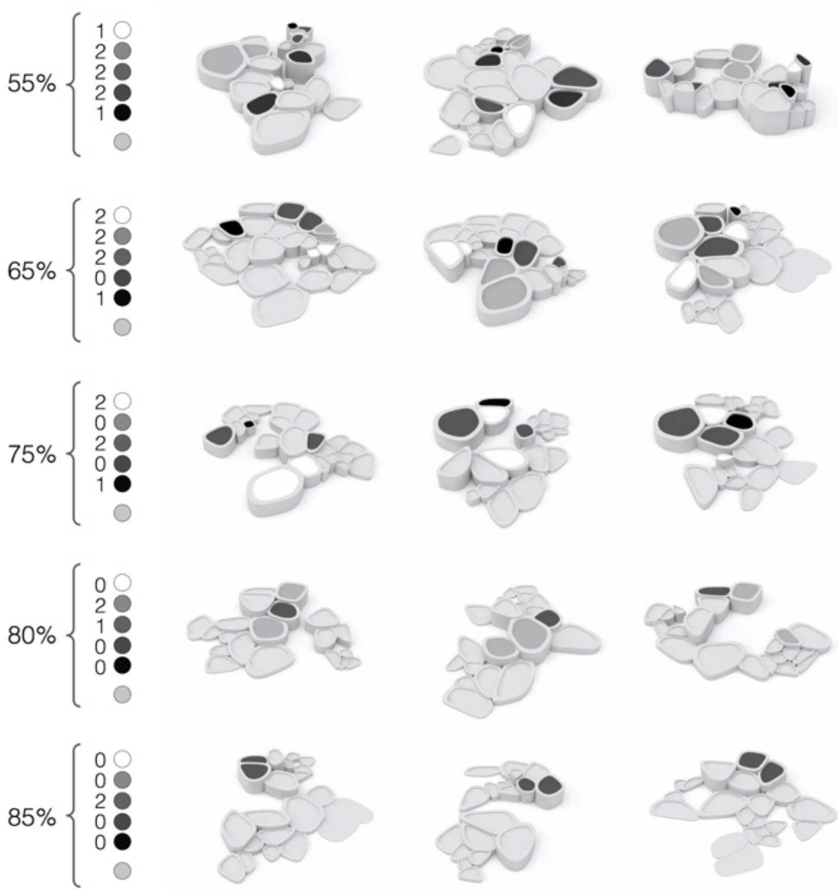
https://www.hindawi.com/journals/ijap/2009/624035/fig5/

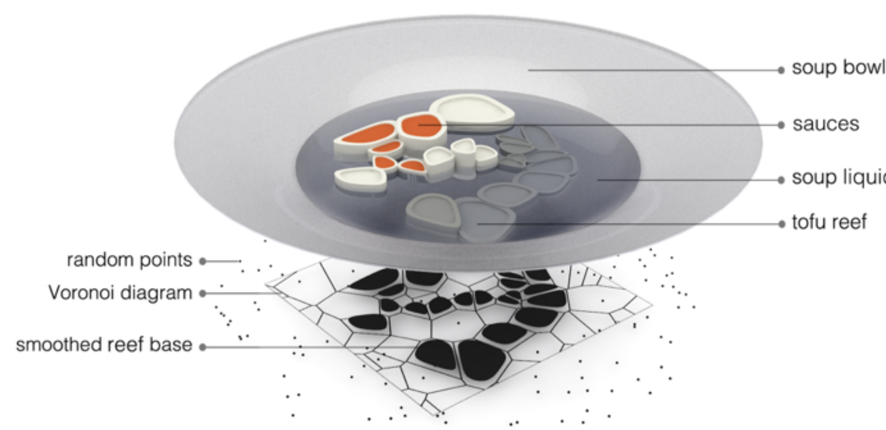
Voronoi Diagram - applications

Neri Oxman / MIT Media Lab

https://decoracion2.com/vuzzle-chair-un-sillon-desmontable/18005/

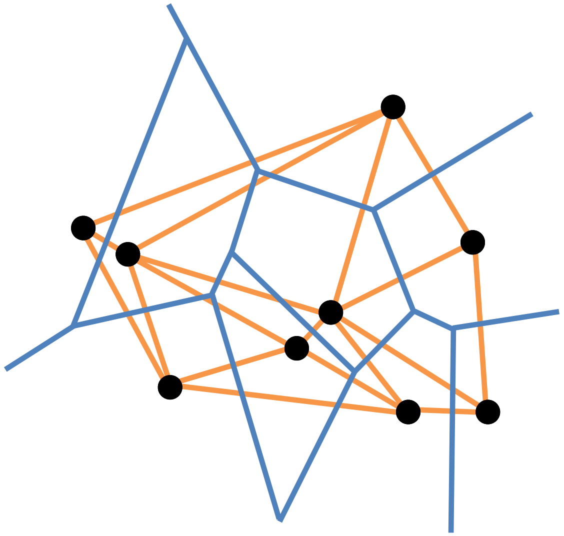
Voronoi Diagram - applications





Delaunay Triangulation

A Delaunay triangulation for a set P of points in a plane is a triangulation $\mathrm{DT}(P)$ such that no point in P is inside the circumcircle of any triangle in $\mathrm{DT}(P)$. Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation. For a set of points on the same line there is no Delaunay triangulation. For four or more points on the same circle the Delaunay triangulation is not unique: each of the two possible triangulations that split the quadrangle into two triangles satisfies the "Delaunay condition".



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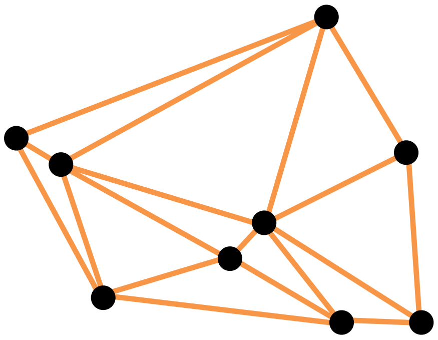


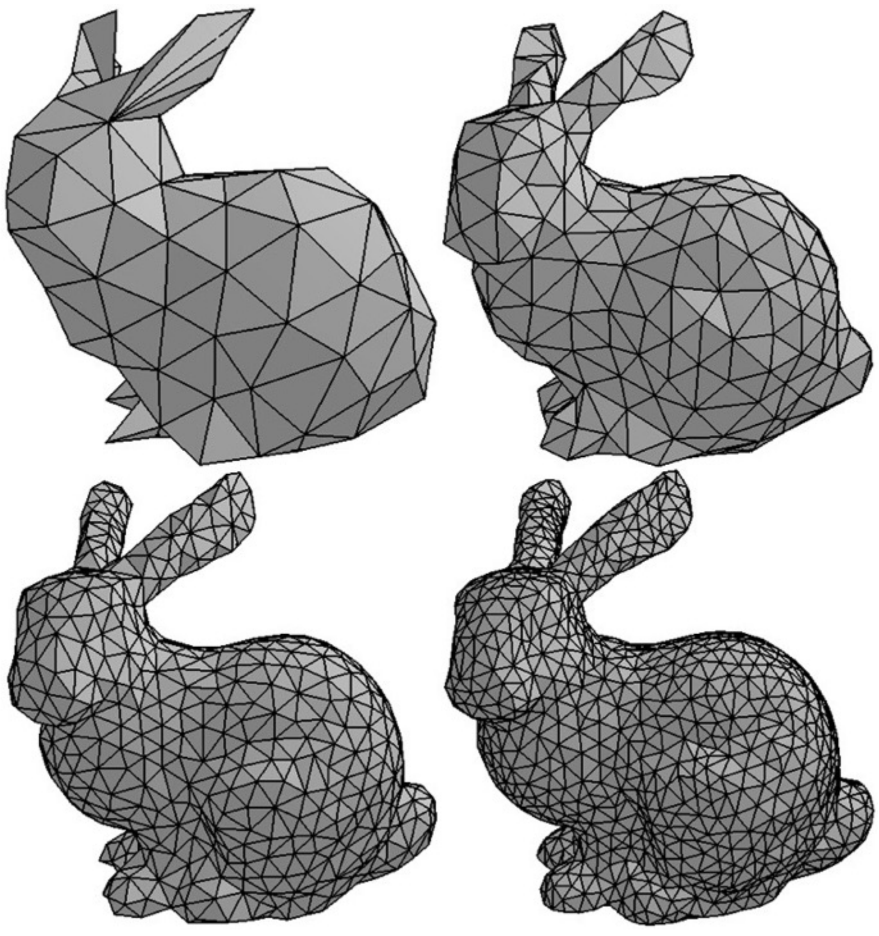
Delaunay Triangulation - some properties

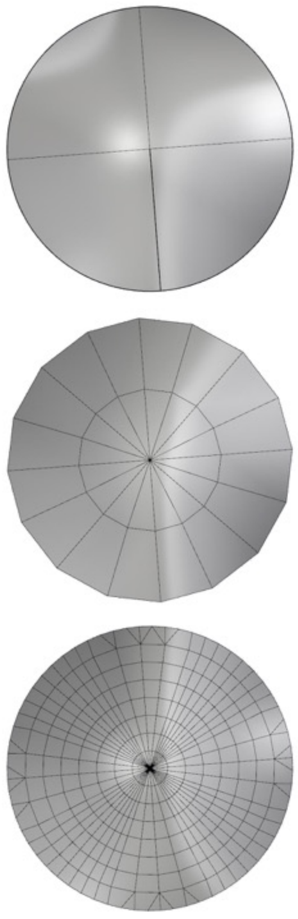
- The union of all simplices in the triangulation is the convex hull of the points.
- In the plane, each vertex has on average six surrounding triangles.
- In the plane, the Delaunay triangulation maximizes the minimum angle. Compared to any other triangulation of the points, the smallest angle in the Delaunay triangulation is at least as large as the smallest angle in any other.
- A circle circumscribing any Delaunay triangle does not contain any other input points in its interior.
- Delaunay triangulation can be obtained from Voronoi diagram by connecting each pair of adjacent points => $O(n \log n)$

Delaunay Triangulation - applications

abduzeedo.com

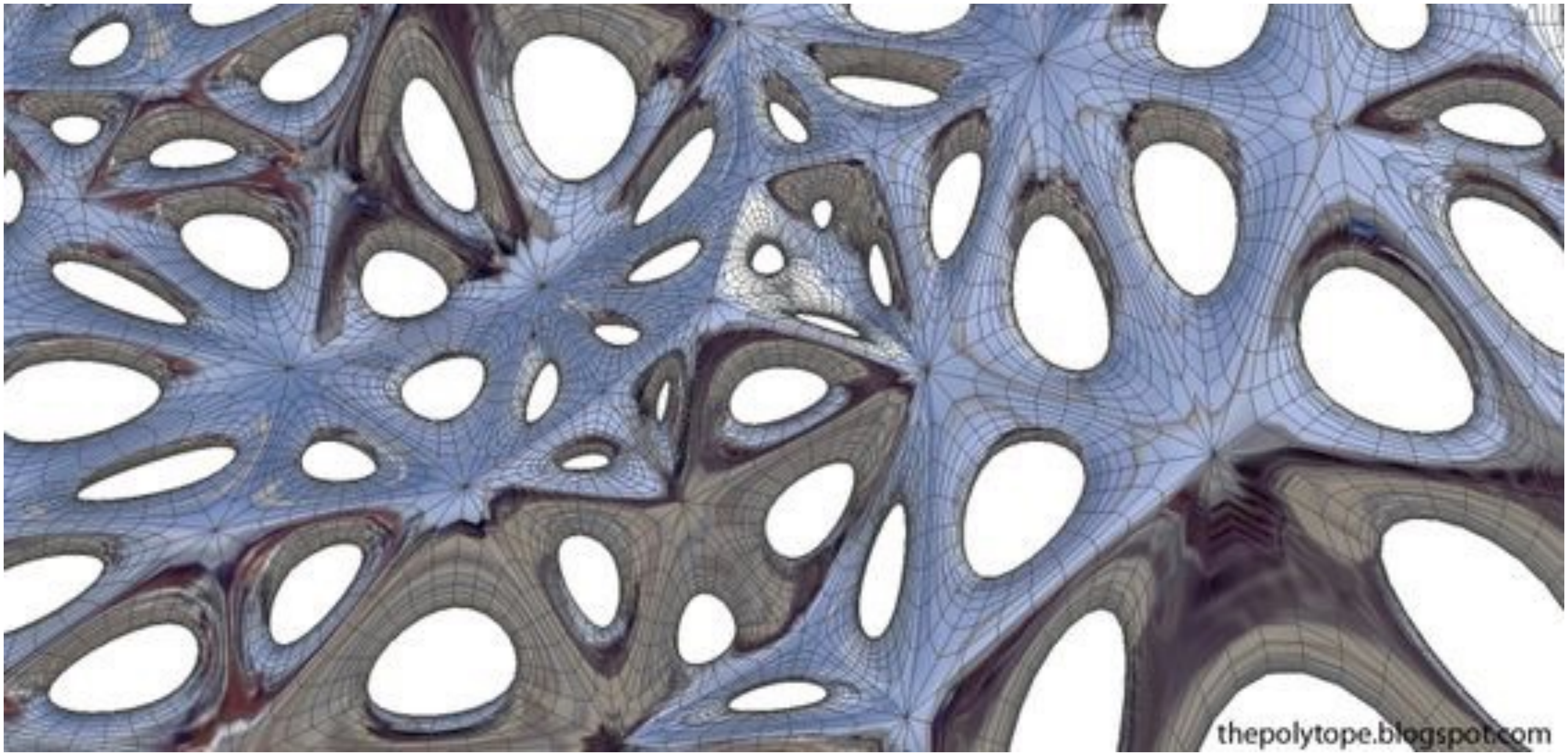
Delaunay Triangulation - applications





Geodesic Computations on 3D Meshes Gabriel Peyré Laurent Cohen

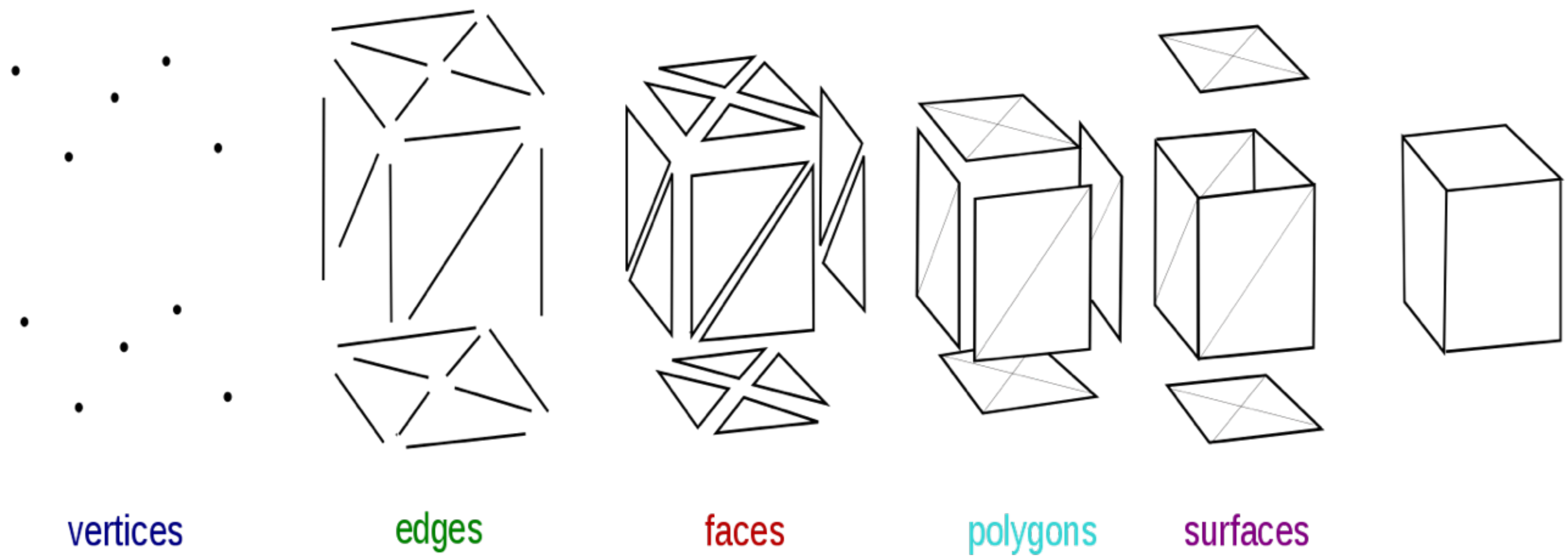
Delaunay Triangulation - applications



Kamil Kiendzierski @ grasshopper3d.com

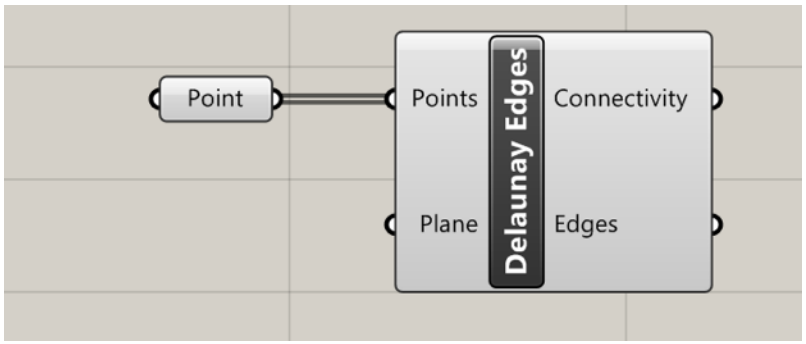
Delaunay Triangulation - applications

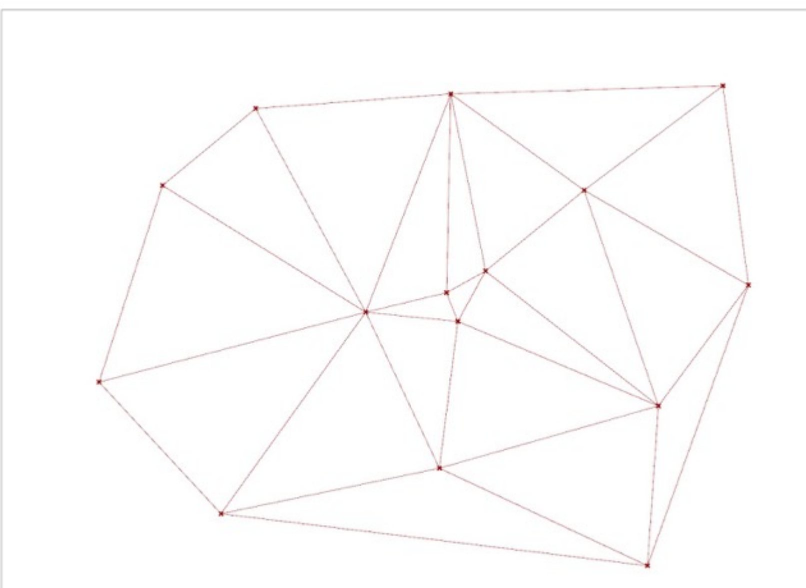
http://n-e-r-v-o-u-s.com/



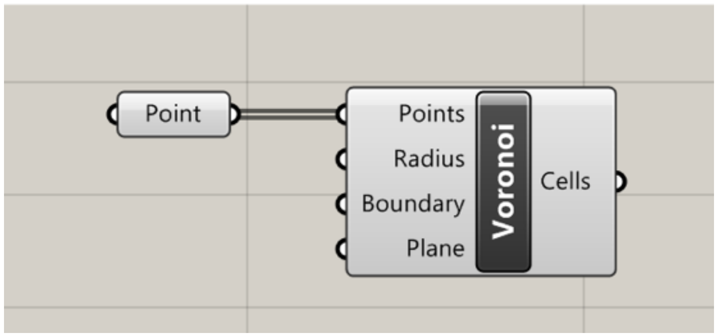
wikipedia

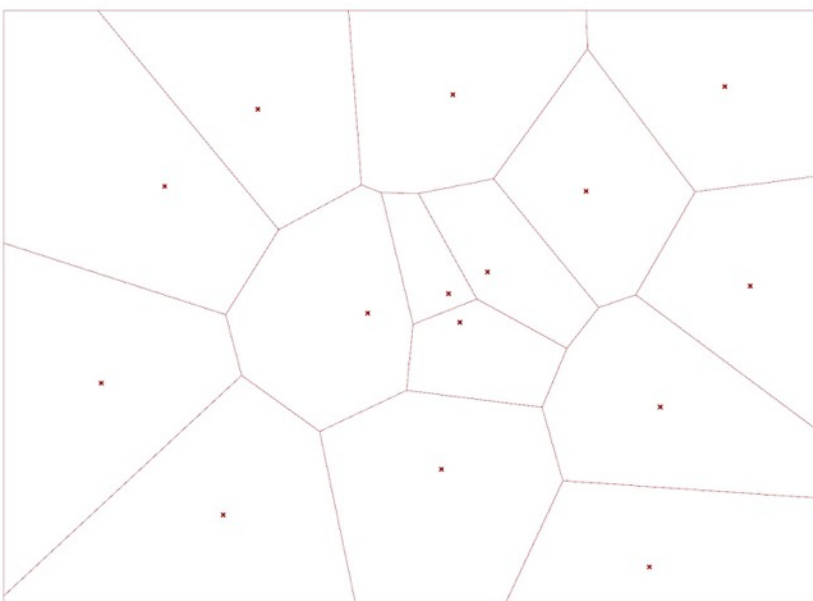
Voronoi / Delaunay - Grasshopper





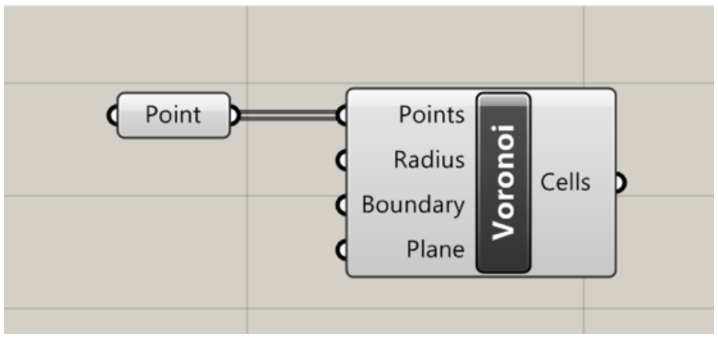
Voronoi / Delaunay - Grasshopper

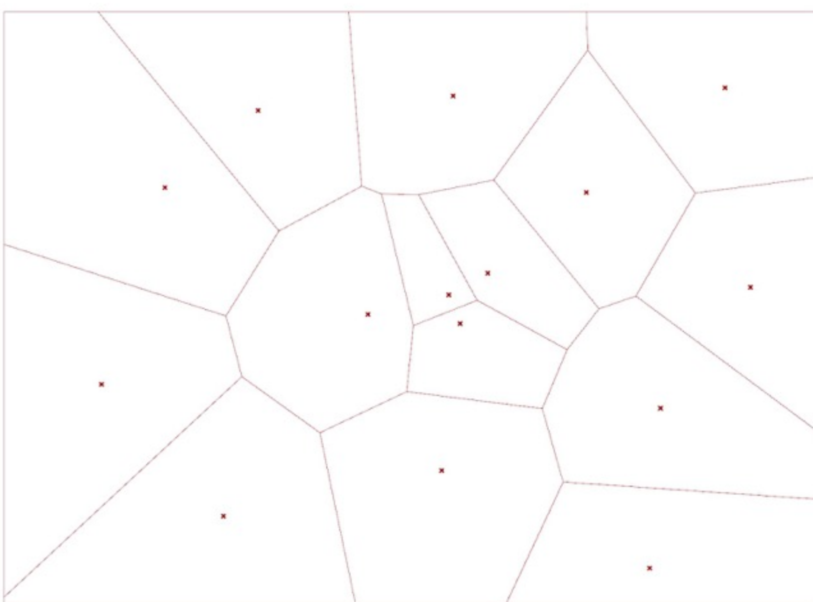




Voronoi / Delaunay - Grasshopper

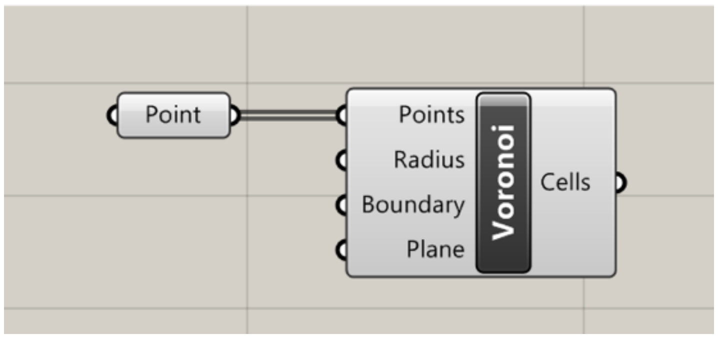
Points distribution





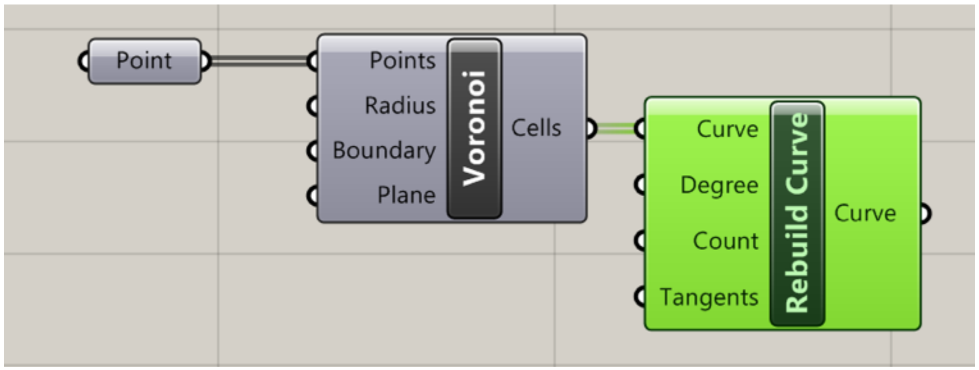
Voronoi / Delaunay - Grasshopper

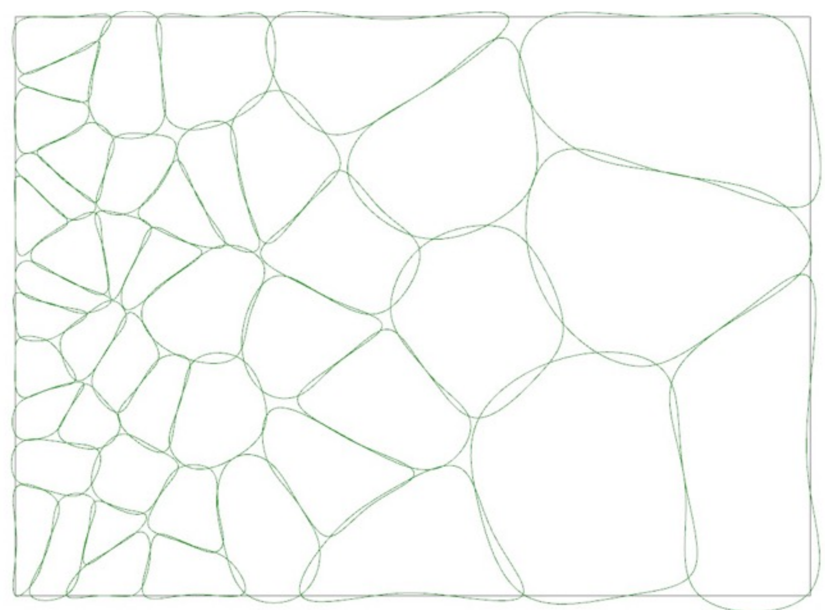
Smoothing corners



Voronoi / Delaunay - Grasshopper

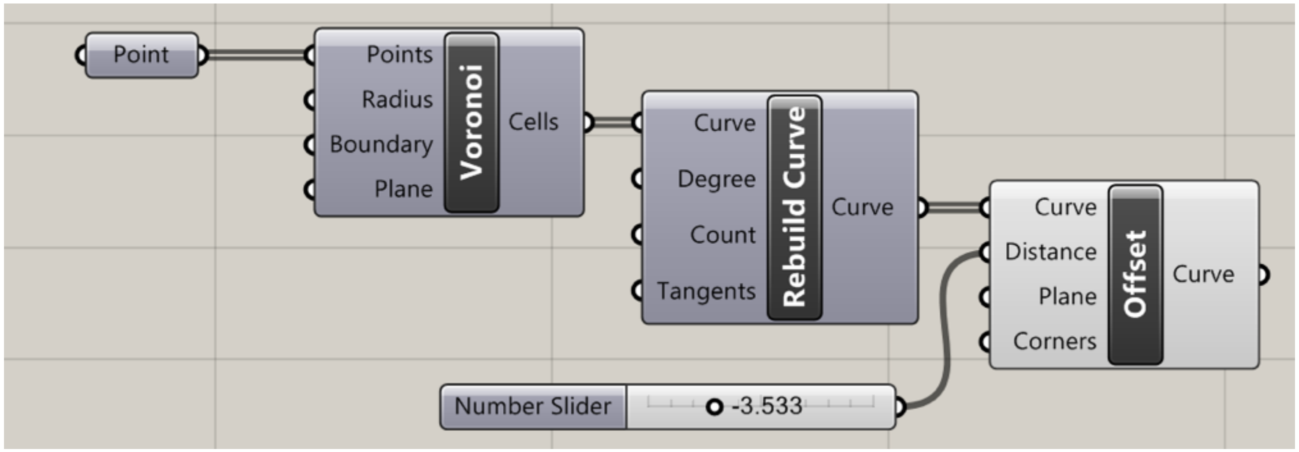
Smoothing corners

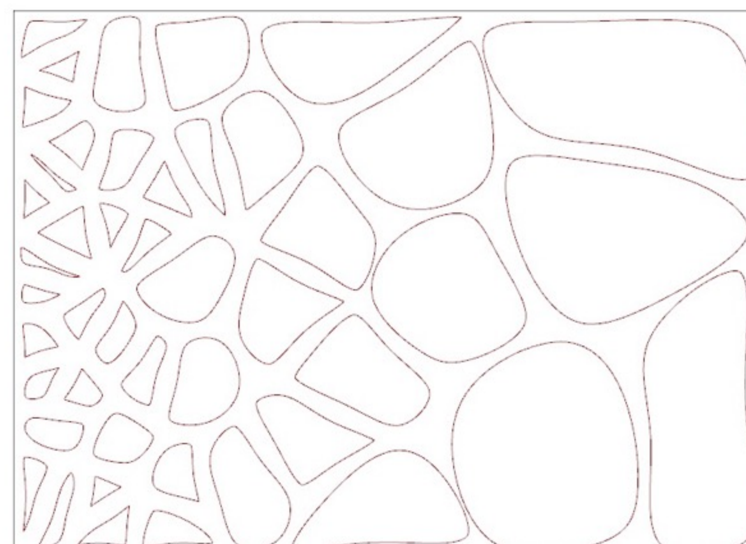




Voronoi / Delaunay - Grasshopper

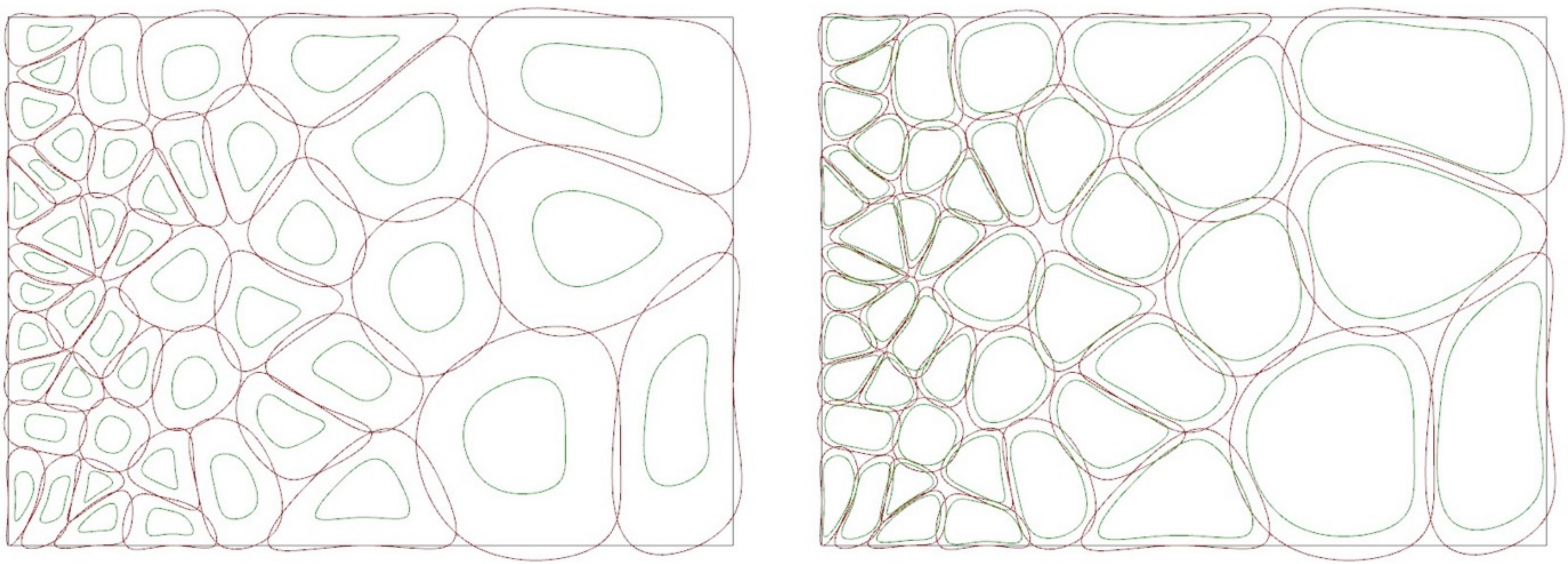
Smoothing corners - **offset**





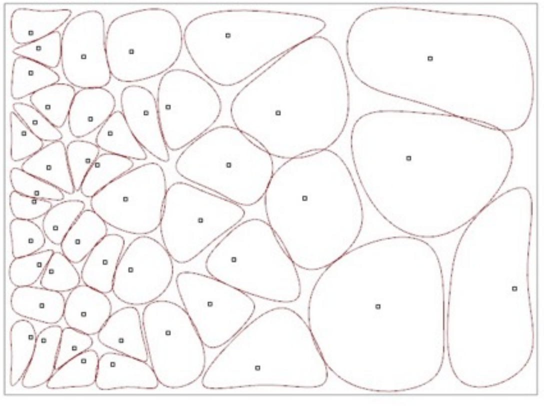
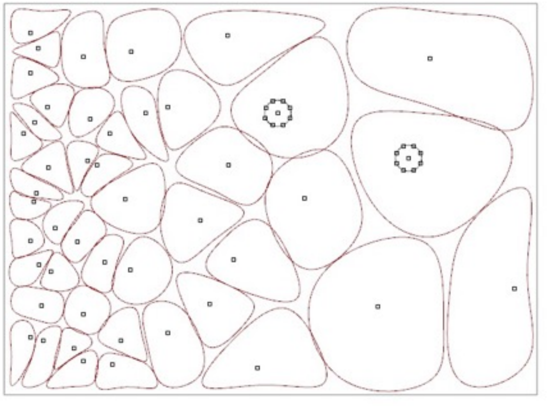
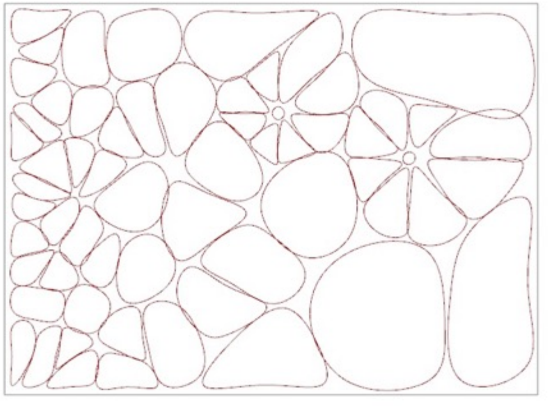
Voronoi / Delaunay - Grasshopper

Curve offset vs. **scale**



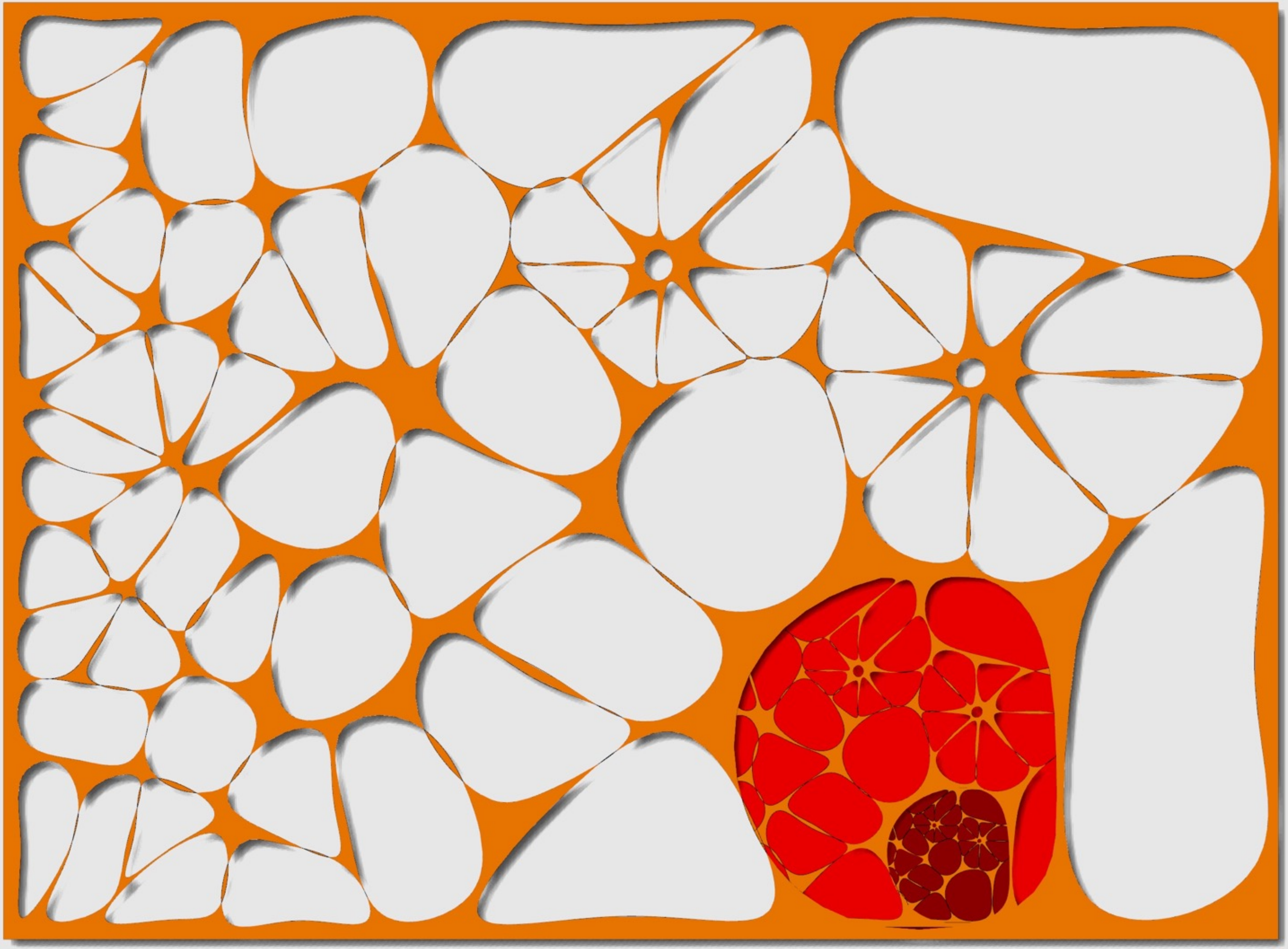
Voronoi / Delaunay - Grasshopper

Embedding symmetry

Voronoi / Delaunay - Grasshopper

2.5D - Curves Loft



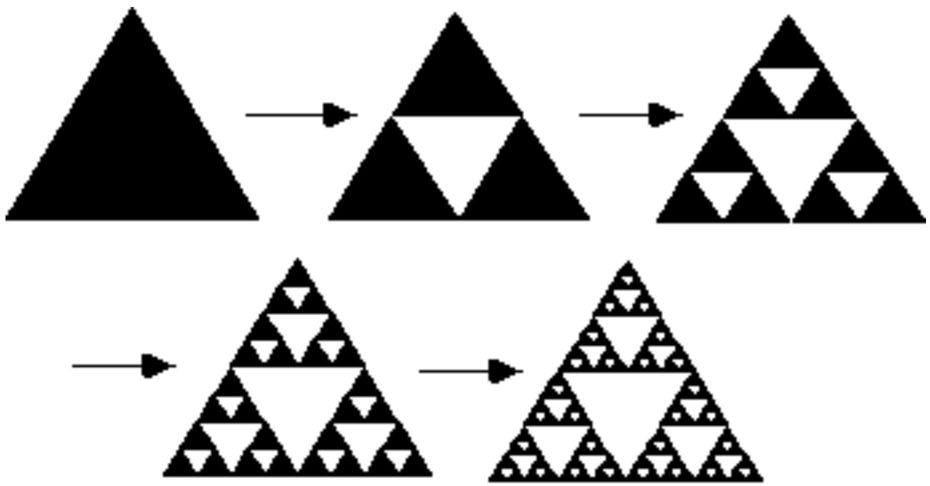
Fractals

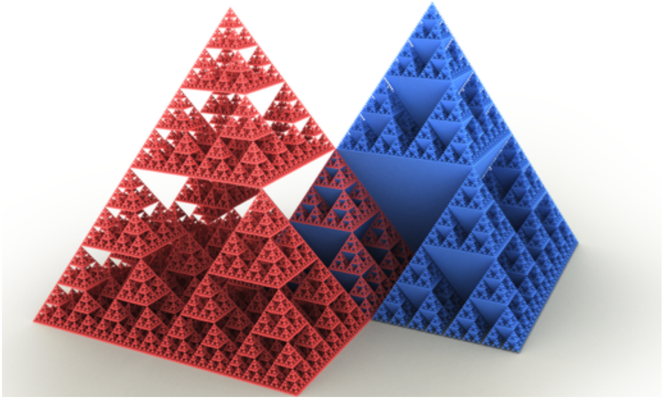
Fractal

A fractal is a mathematical set that exhibits a repeating pattern displayed at every scale. It is also known as expanding symmetry or evolving symmetry. If the replication is exactly the same at every scale, it is called a self-similar pattern. Fractals can also be nearly the same at different levels.

Fractal - famous examples

- **Sierpinski Triangle**

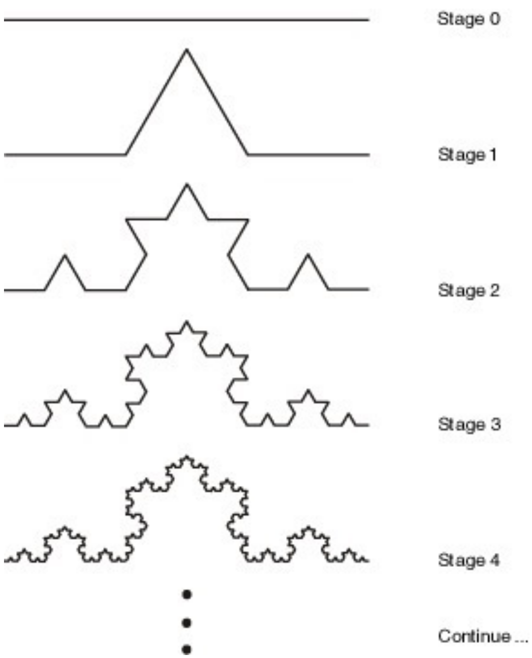


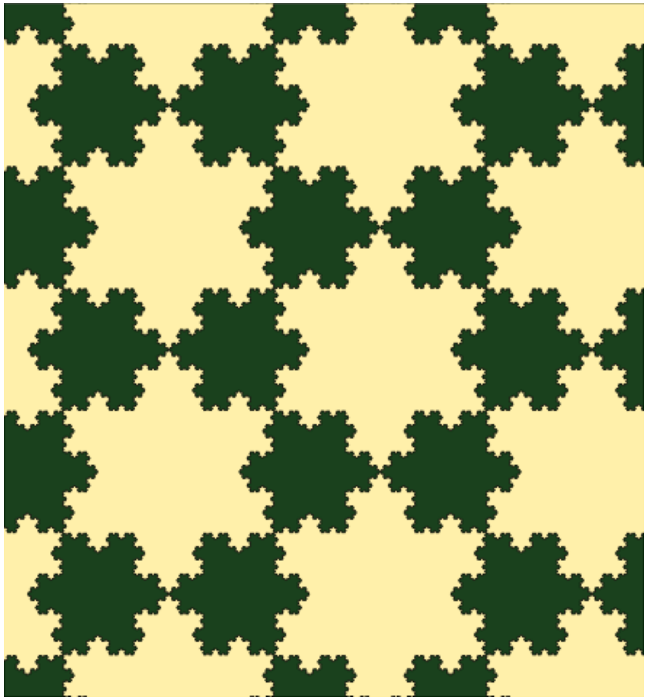


http://math.bu.edu/DYSYS/chaos-game/node2.html

Fractal - famous examples

- **Von Koch Curve**





https://adonis49.wordpress.com/tag/von-koch-curve/

Fractal - realism

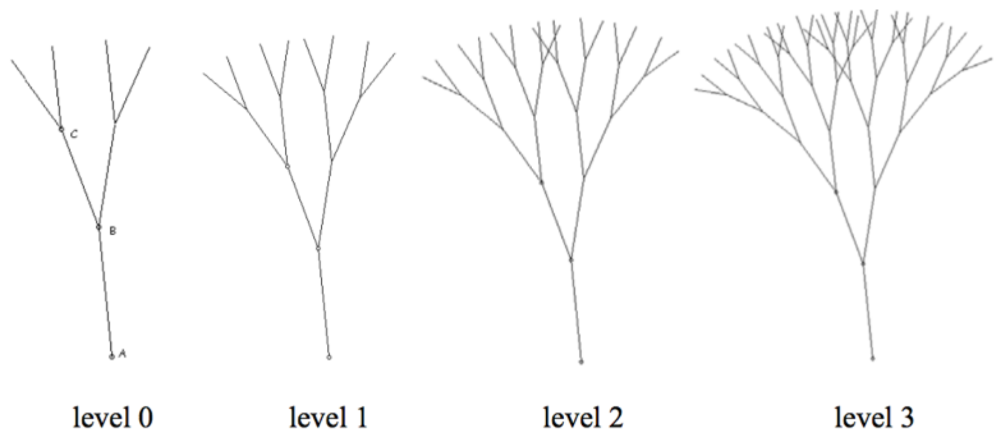
A very important application of fractal is to reproduce the natural images such as clouds, trees, mountains and etc. This is because many natural things, such as plants, are very complex and exhibit some self-similarity. The complexity of fractals and the their property of self-similarity allows fractals to reproduce a large set of real-world images.

In fact, the Koch Snowflake we discussed before is already a possible starting point for the design complex natural curves. The Koch Snowflake itself is already very similar to the snowflake in the natural world. However, the Koch Snowflake does not look like a natural curve. It is too regular in shape. This regularity comes from the strictness of its construction process.

http://www.math.nus.edu.sg/aslaksen/gem-projects/maa/World_of_Fractal.pdf

Fractal - realism

Original Binary Tree Fractal



Modified Binary Tree Fractal

http://www.math.nus.edu.sg/aslaksen/gem-projects/maa/World_of_Fractal.pdf

Common techniques for generating fractals

- Iterated function systems – use fixed geometric replacement rules; may be stochastic or deterministic.
- Strange attractors – use iterations of a map or solutions of a system of initial-value differential or difference equations that exhibit chaos.
- L-systems – use string rewriting; may resemble branching patterns, such as in plants, biological cells.
- Escape-time fractals – use a formula or recurrence relation at each point in a space
- Random fractals – use stochastic rules.
- Finite subdivision rules - use a recursive topological algorithm for refining tilings and they are similar to the process of cell division.

Fractals - nature

https://www.wired.com/2010/09/fractal-patterns-in-nature/

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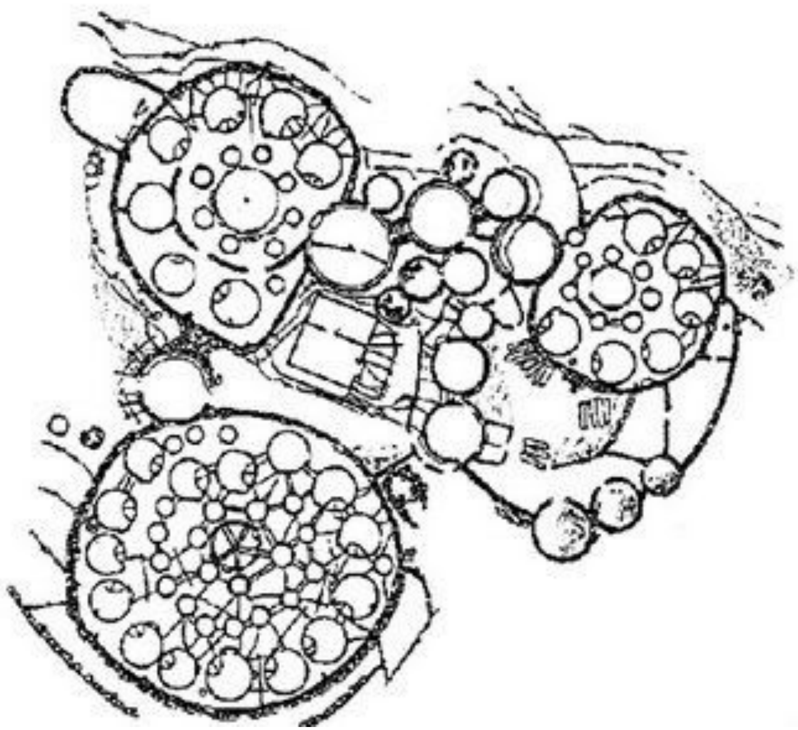
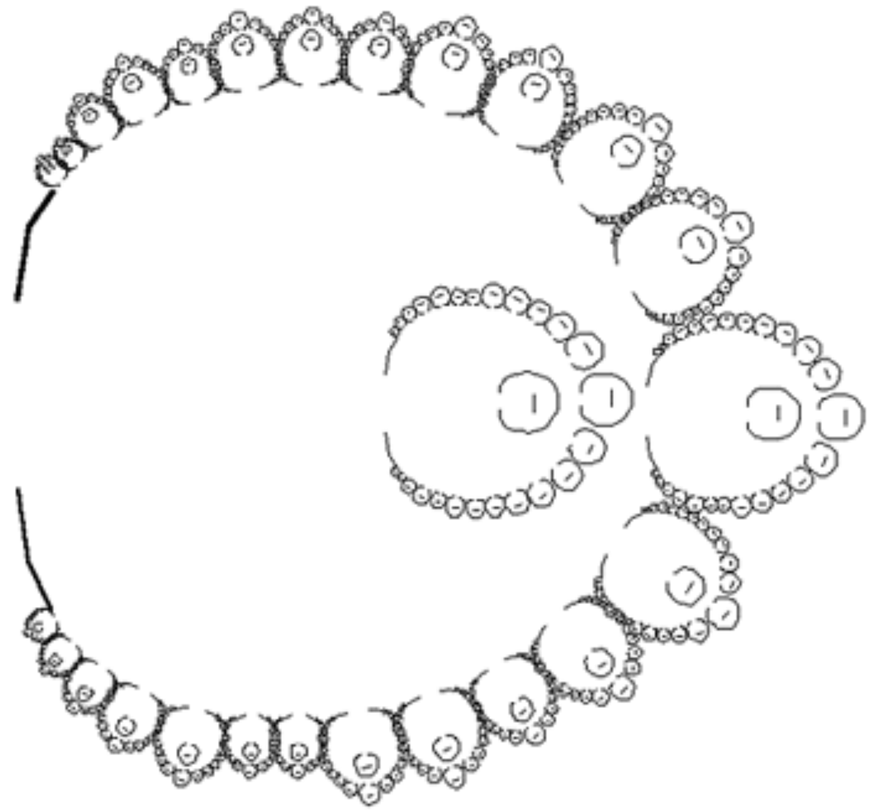
https://www.wired.com/2010/09/fractal-patterns-in-nature/

Fractals - applications



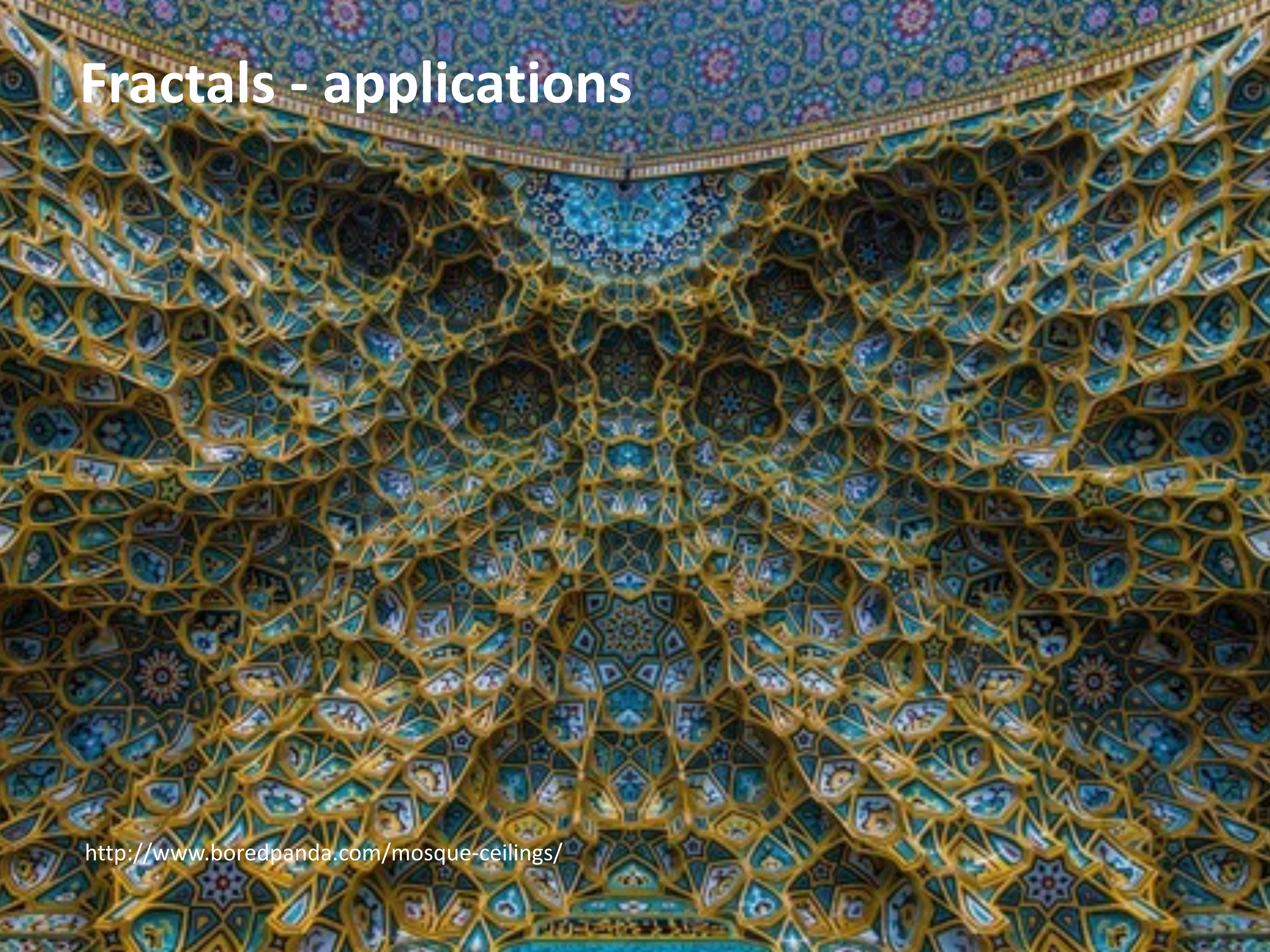


http://www.ted.com/talks/ron_eglash_on_african_fractals.html

Fractals - applications

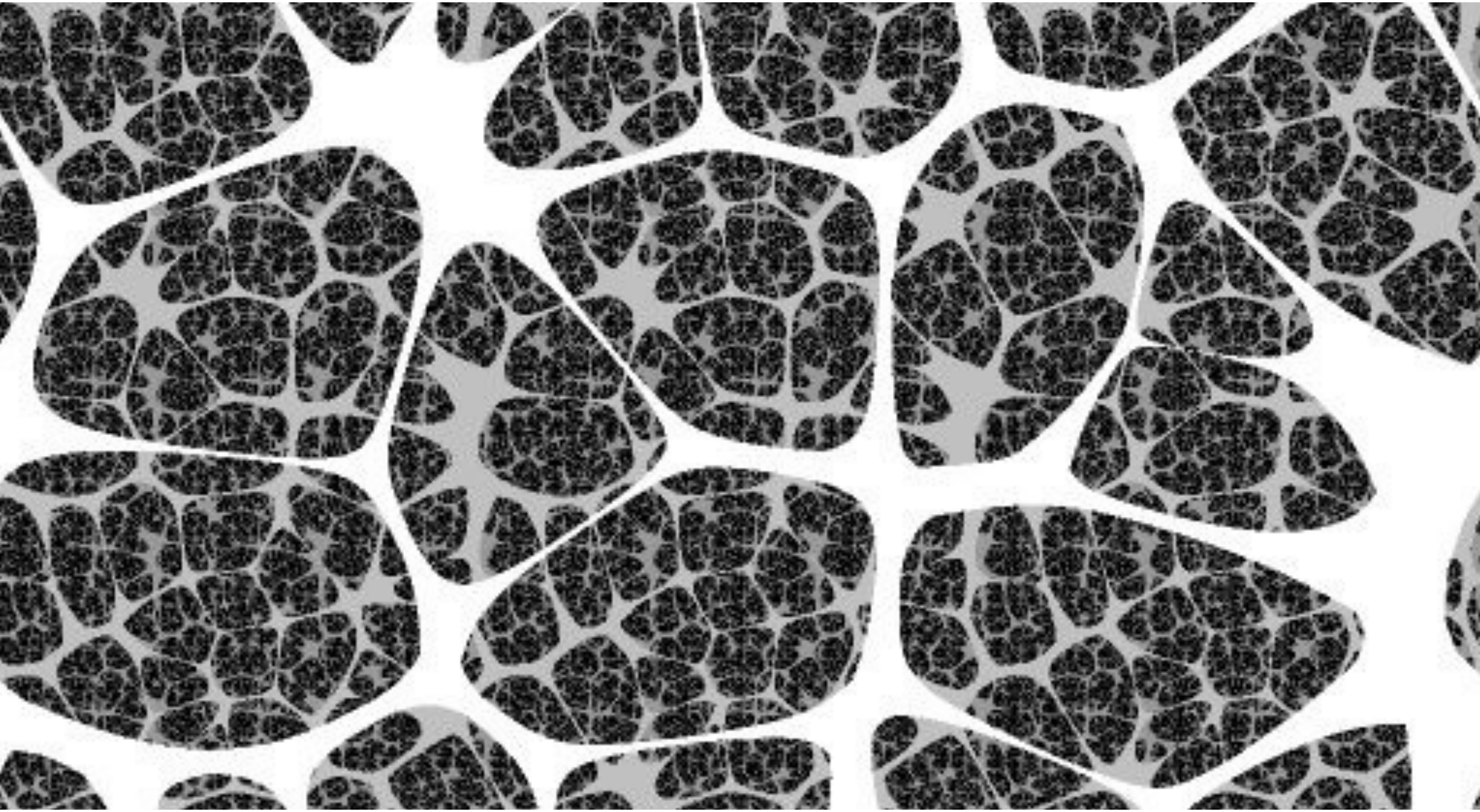
http://happinessinmay.com/2013/09/16/fractals-in-architecture-and-fashion-antoni-gaudi-peter-pilotto/

Fractals - applications



http://www.boredpanda.com/mosque-ceilings/

Fractals - applications

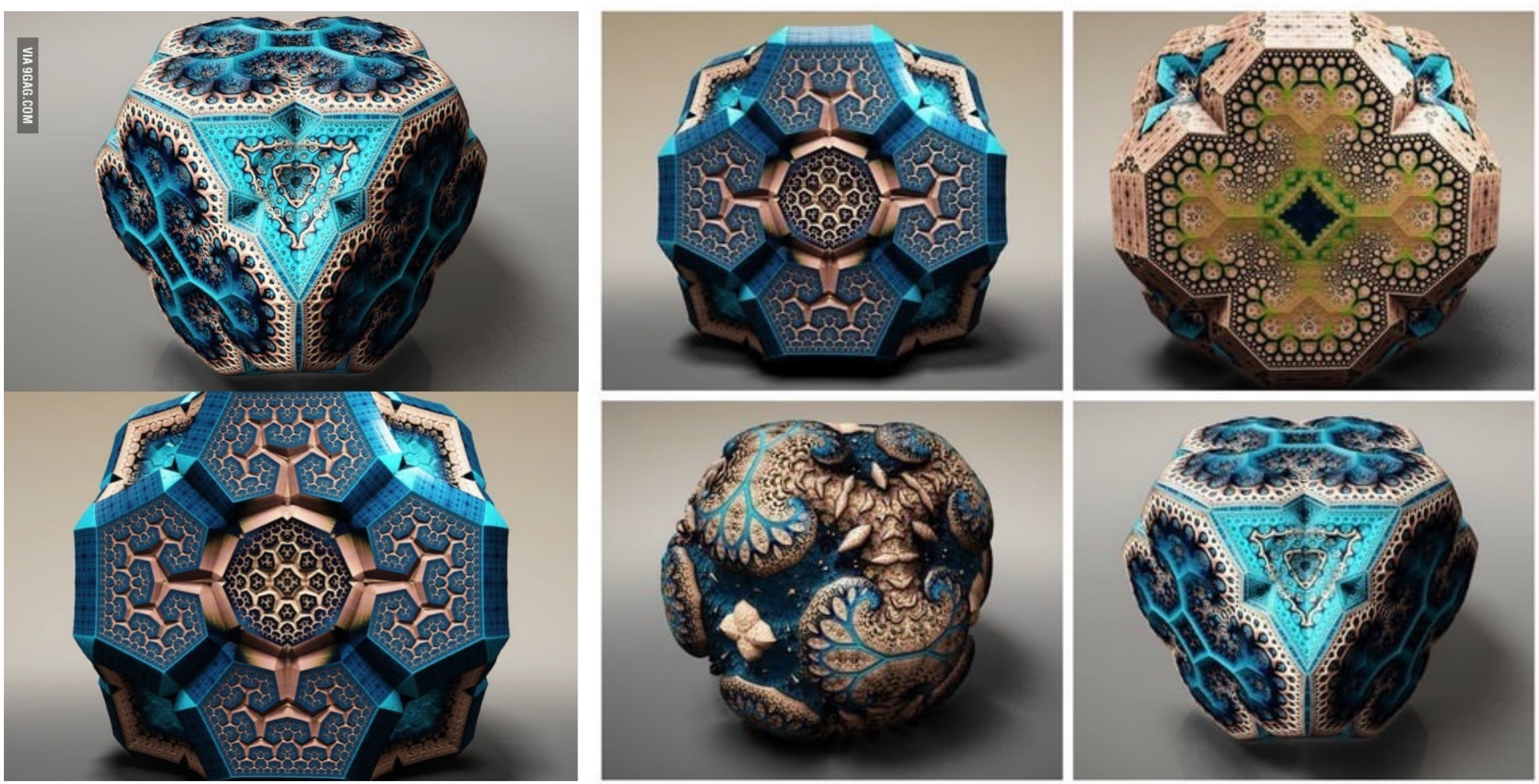


https://www.pinterest.com

Fractals - applications

http://en.academic.ru/dic.nsf/enwiki/30071

Fractals - applications



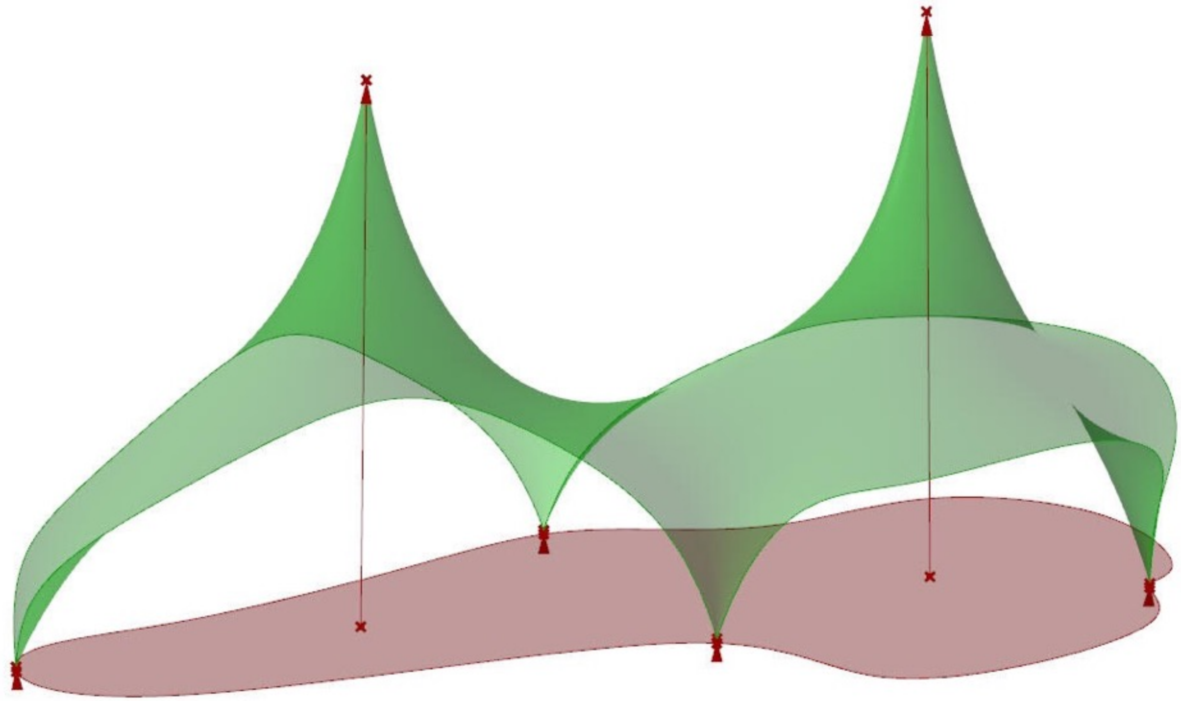
http://indulgy.com/post/BUNEFCrBV2/faberg-fractals

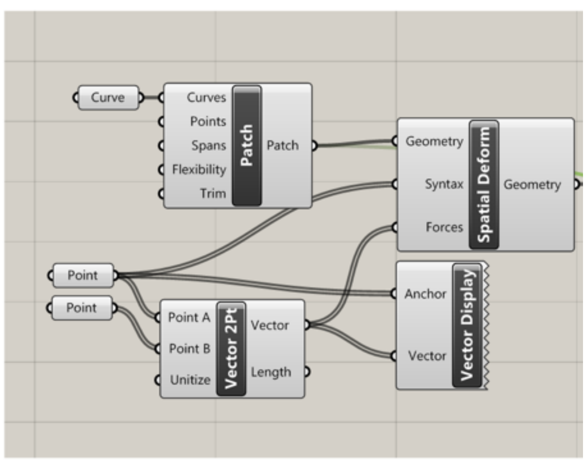
Fractals - applications

https://pixabay.com/en/fractal-menger-sponge-cuboid-1128622/

Projection

Projection

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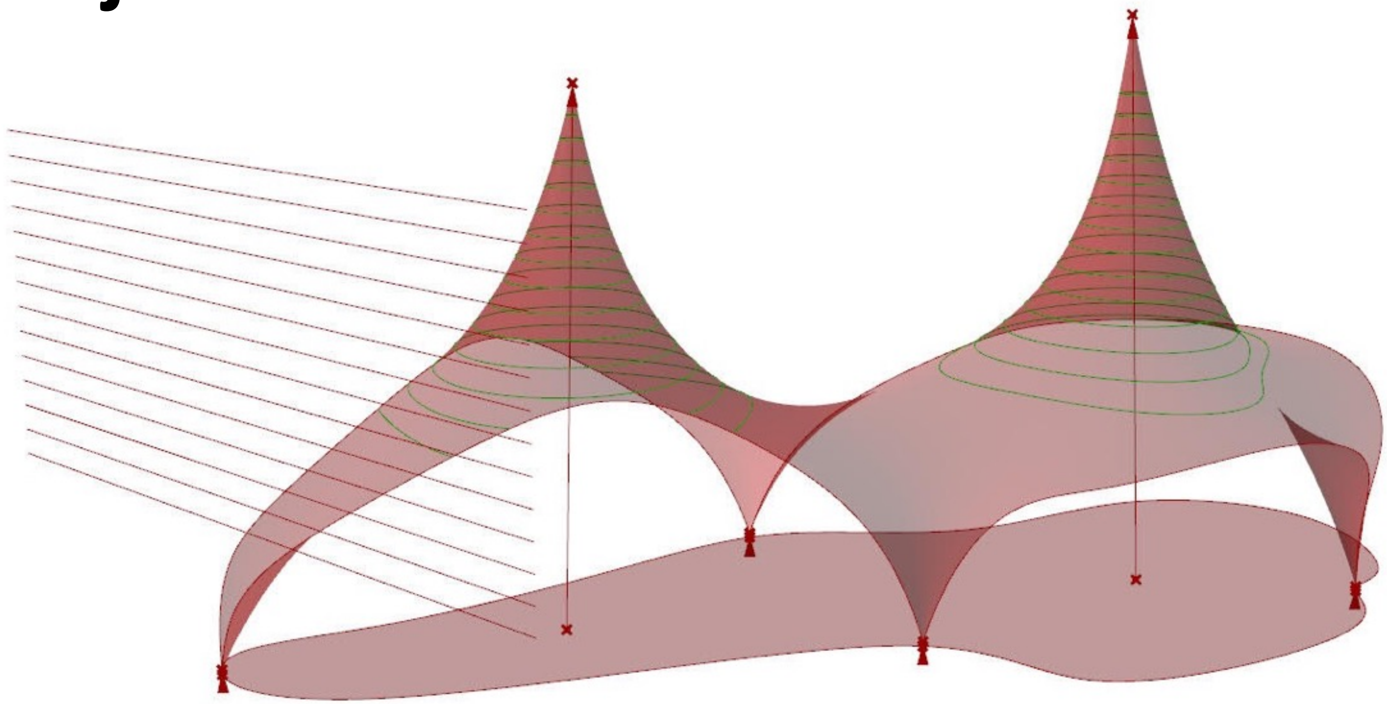
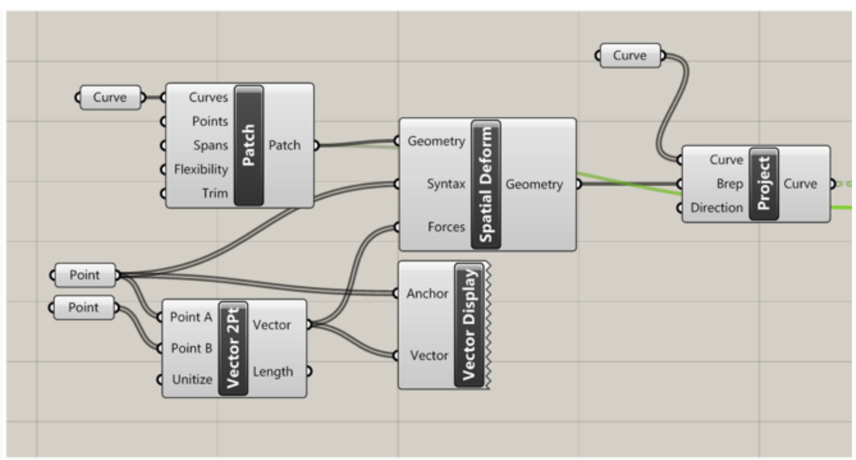
Projection

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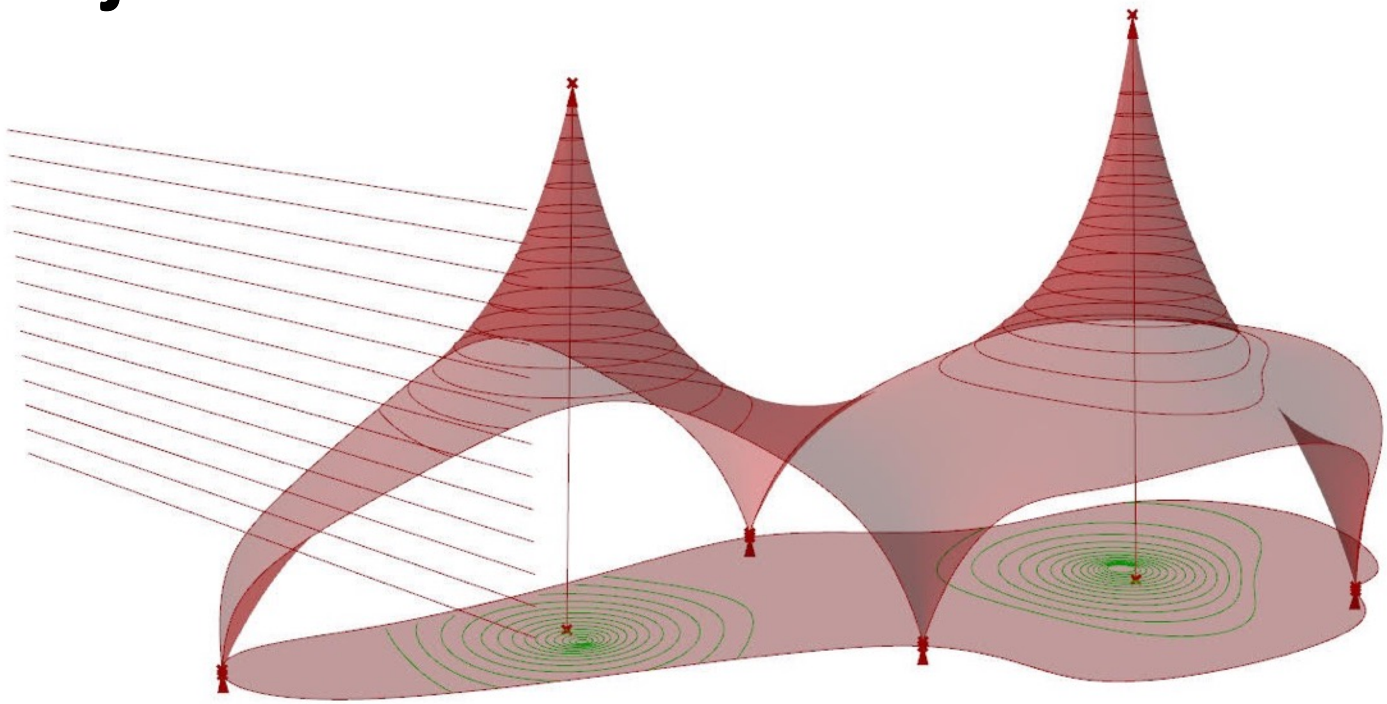
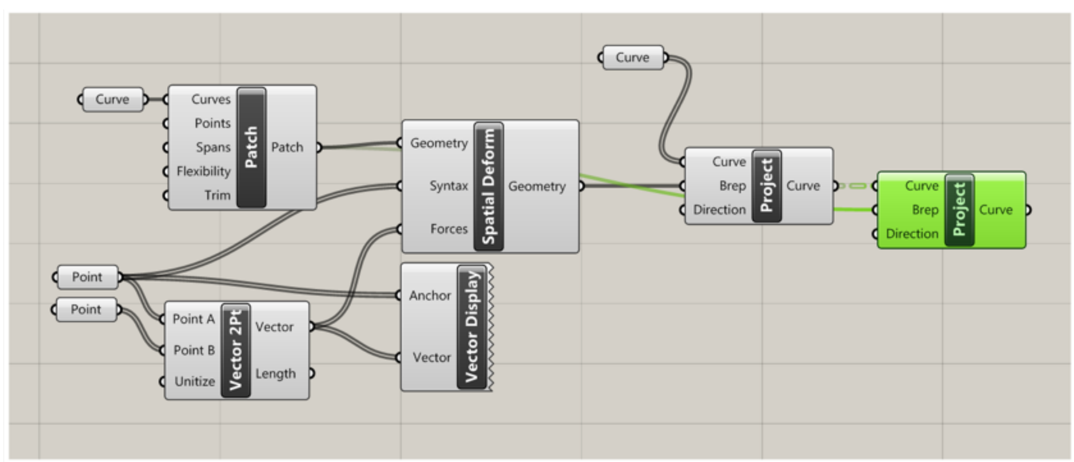
Projection

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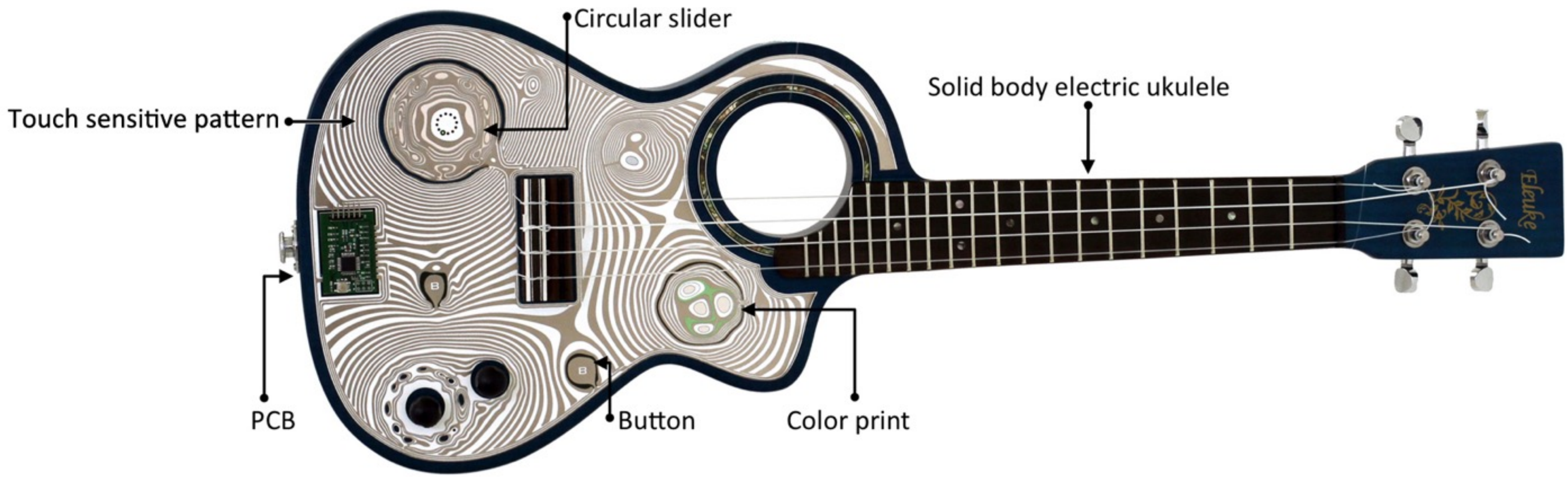
Projection

Projection

(a) A photo of a Z-Guard prototype with five proximity sensors and two press buttons (marked with "P"), assembled on a real electric guitar (Squier Stratocaster). (b) An illustration of the design with silver and gold masks to visually distinguish the sensing areas.

Z-Guard: Printed Circuit Boards as Interactive Guitar Pick Guards with Nan-Wei Gong

Projection



Inkjet-printed Conductive Patterns for Physical Manipulation of Audio Signals
with Nan-Wei Gong and Joe Paradiso